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Non-local Robin type boundary conditions for use in wall functions

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Introduction

Non-local wall functions (Utyuzhnikov, 2009) are a new type of wall function that attempt to account for source terms and convection terms across the viscous sublayer in a flow. In the one-dimensional case, they can be considered as a generalisation of the analytical wall function of Craft et al. (2002). This results in Robin type boundary conditions of the form $U = \alpha + \beta \partial_y U$ (Utyuzhnikov, 2012). The only quantity needed in these wall functions is a specification of the turbulent viscosity across the near-wall layer. Currently implemented in RANS with the $k - \varepsilon$ model, it may be possible to extend them to LES.

These boundary conditions can be transformed into non-local wall functions with a domain decomposition method. With these, simplified, linear problems are solved in the boundary layers of a flow, which allows non-local effects to be included in the boundary conditions.

Theory

- The $k - \varepsilon$ boundary layer equations can be written for U, k and ε in the form

$$\partial_y \left((\mu + \mu_t) \partial_y U \right) = R_{hu}.$$

- The left hand side contains only diffusion to the wall. The right hand side, R_{hu}, contains all other terms, e.g. pressure gradient, modelled convection terms.
- Robin boundary conditions are obtained by integrating twice: once from y_P to y and again from 0 to y. This yields

$$U_P = U_{\text{wall}} + f_1 \left. \frac{\partial U}{\partial y} \right|_P - \frac{f_2}{y_P (\mu + \mu_t)_P} \int_0^{y_P} R_{hu} \, dy, \qquad (1)$$

$$f_1 = \int_0^{y_P} \frac{(\mu + \mu_t)_P}{\mu + \mu_t(y)} \, dy, \qquad (2)$$

$$f_2 = y_P \int_0^{y_P} \frac{\mu_P}{\mu + \mu_t(y)} \left(1 - \frac{\int_0^y R_{hu} \, dy'}{\int_0^{y_P} R_{hu} \, dy'} \right) dy. \qquad (3)$$

- The only free parameter is the turbulent viscosity across the sublayer, $\mu_t(y)$.
- The right hand side terms can be modelled, e.g. for $R_{hk} = -\rho P_k + \rho \varepsilon$
- $P_k = \nu_t (\partial_y U)^2$, where $\partial_y U$ can be calculated using Equation 1, and
- $\varepsilon(y) = \min \left(\frac{2\nu k_P}{y_v^2}, \frac{k_P^{3/2}}{c_l y} \right)$.

1-D channel flow results

- The first test case was a channel flow at $Re_\tau = 590$ (Moser et al., 1999).

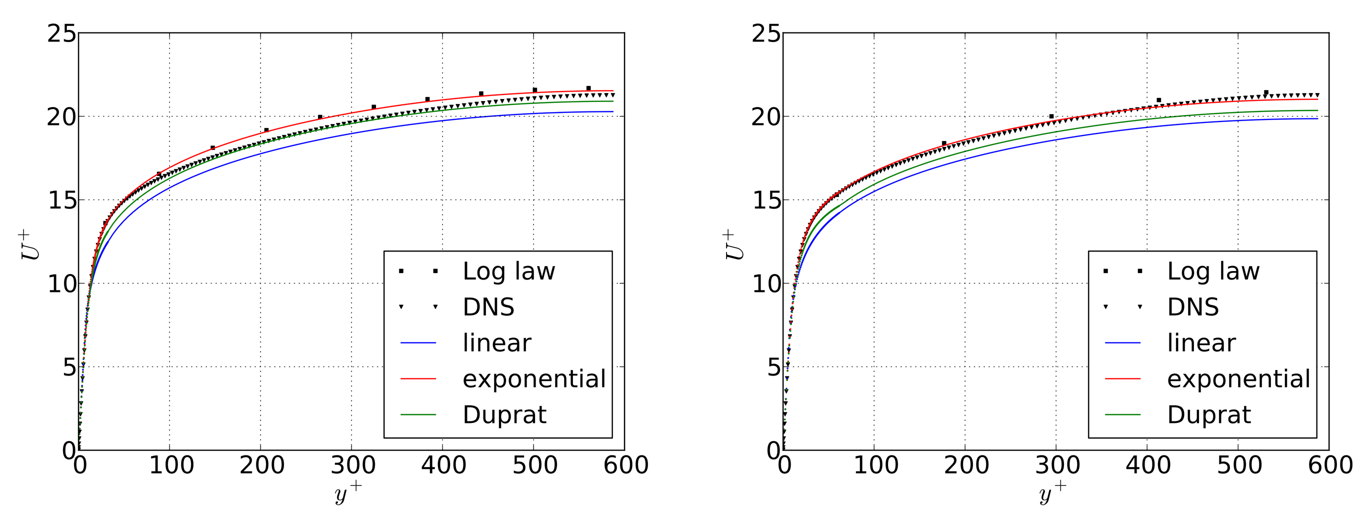

Figure 1: (a) $y_P = 0.05h$ (b) $y_P = 0.1h$.

- Results are competitive with the standard *Code_Saturne* wall functions.

Non-local wall functions (NLWF)

- With these, the boundary layers can now be simulated in advance of the bulk.
- Two (or more), linear problems are solved, for W_e and W_0 over the sublayer

$$\partial_y \left((\mu + \mu_t) \partial_y W_e \right) = R_{hu}, \quad W_e(y_P) = 0, \qquad \text{and} \qquad \partial_y \left((\mu + \mu_t) \partial_y W_0 \right) = 0, \quad W_0(y_P) = 1,$$

- The flow across the boundary layer depends on the velocity at y_P

$$W(y) = W_e(y) + U(y_P) W_0(y),$$
$$\partial_y W(y) = \partial_y W_e(y) + U(y_P) \partial_y W_0.$$

- A good approximation of τ_w is required in order to specify $\mu_t(y)$. Without this, the two simulations must be coupled and run toghether.
- k retains its 1-D Robin type boundary condition, as before.

NLWF results on a 1-D channel flow

- The test case is a channel flow at $Re_{\text{bulk}} = 11 \times 10^3$. The exponential viscosity profile has been used.

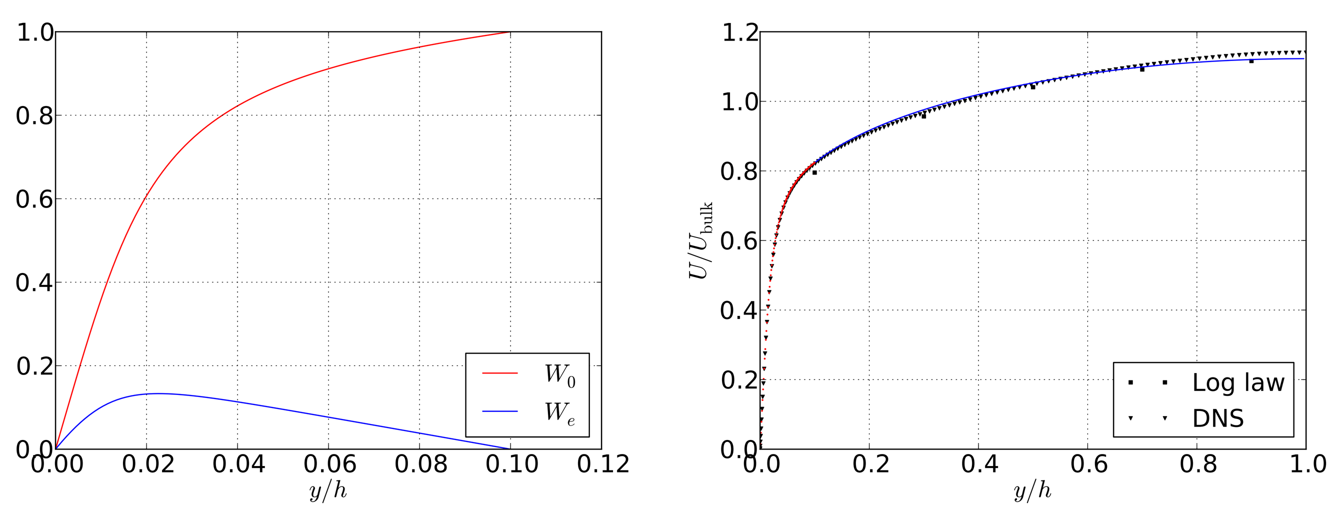

Figure 3: (a) W_0 and W_e (b) U profile with $y_P = 0.1$.

- The results are in excellent agreement with the DNS data.

Turbulent viscosity profiles

- Three viscosity profiles have been trialled
- A piecewise linear profile, of Craft et al. (2002):

$$\mu_t = \mu c_\mu c_l (y^* - y_v^*), \text{ if } y > y_v$$

- An exponential profile, of Cabot and Moin (1999):

$$\mu_t = \mu \kappa y^+ \left(1 - \exp(-y^+/A) \right)^2$$

- The profile of Duprat et al. (2011), which accounts for the pressure gradient:

$$\mu_t = \mu \kappa y^* \left[\alpha + y^* (1 - \alpha)^{3/2} \right]^\beta \left[1 - \exp\left(-y^* / (1 + A\alpha^3) \right) \right]^2.$$

Advantages over other wall functions

- Robin type boundary conditions are numerically stable.
- U and $\partial_y U$ can be taken at the same iteration $\implies$ faster convergence.
- No restrictions on $\mu_t(y)$ and R_{hu} because they are integrated numerically.
- Can even use different turbulence models over near-wall region and bulk flow.

1-D annular flow results

- Second test case is a statistically 1-D flow down an annular pipe with $R_2/R_1 = 5.90996$ and $Re_{\text{bulk}} = 4 \times 10^5$.
- The LRN SST model was used as the benchmark solution.

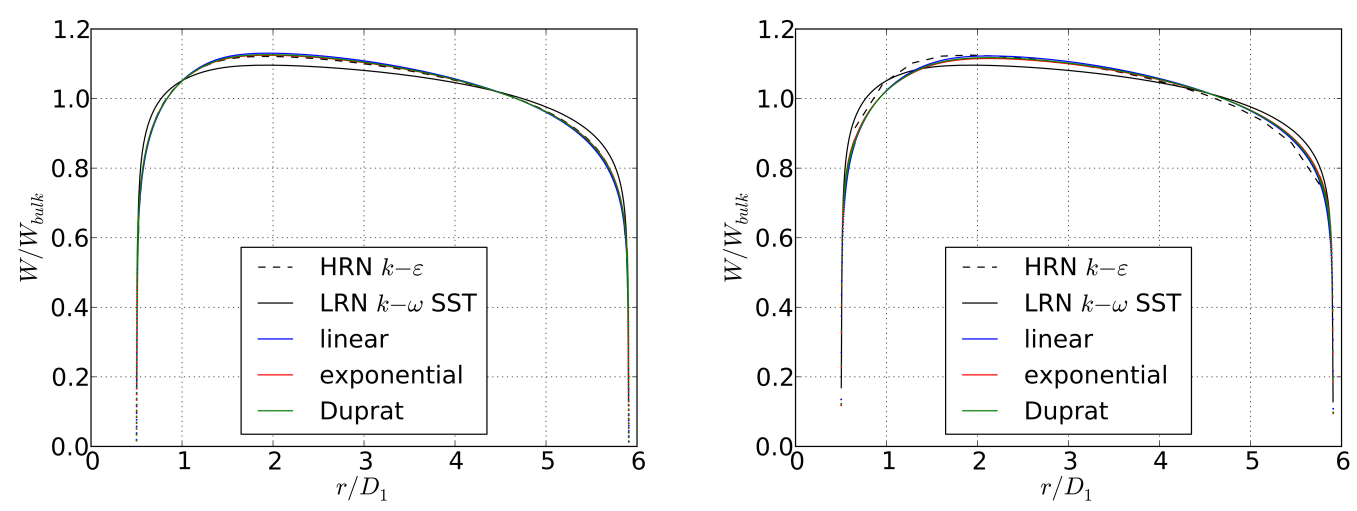

Figure 2: (a) $y_P = 0.04R_1$ (b) $y_P = 0.3R_1$.

Conclusions and further work

Robin type boundary conditions and NLWFs clearly give good results for one dimensional flows. More testing is needed on multidimensional flows with more complex physics. The next stage in this project is to apply non-local wall functions to a complex flow through a diffuser and in a ribbed channel. NLWFs should be able to produce far more accurate results and capture the flow physics better than standard wall functions in these flows. The ultimate goal of the project is to simulate the flow past roughened fuel pins in an AGR fuel assembly.

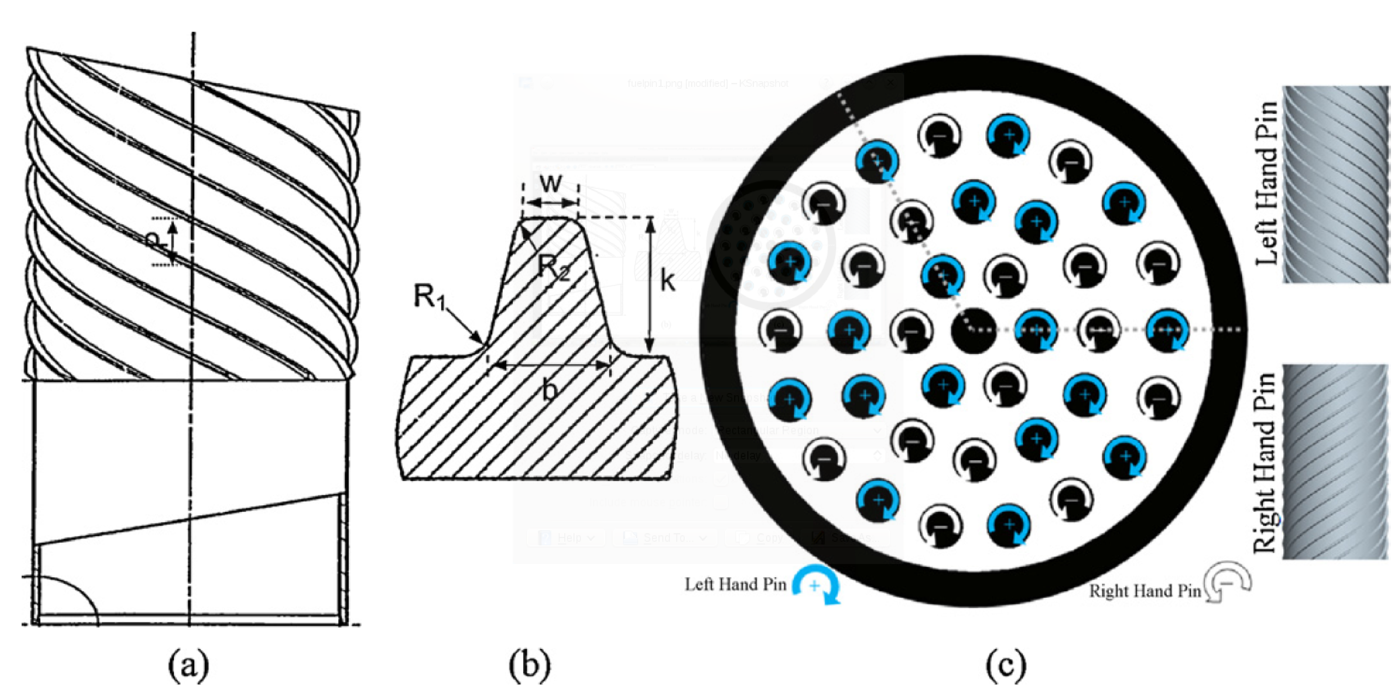

Figure 4: An AGR fuel assembly, from Keshmiri (2011).

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