An improved dissipation rate equation for the $\overline{v^2} - f$ model to account for turbulent transport mechanism in a boundary layer

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Abstract

Since its original introduction in Durbin (1991), more than 15 different versions of the $\overline{v^2} - f$ model have been proposed, the purpose of most of them being to cure the recognised numerical stiffness associated to the original formulation. Even though comparisons of variants show results do not significantly differ from one another on near-wall cases (Laurence et al. (2004), Hanjalić et al. (2004)) a large variability exists in the choice of equations source terms and constants of the resolved turbulent variables, particularly for the dissipation rate ε equation. As it will be seen, this yields difference of behaviour in fundamental flows.

Based on the review of nine of these $\overline{v^2} - f$ variants, with emphasis on the modelling employed for the ε equation, the present work proposes a modification to: 1) reduce the inter-dependance of the equation terms, hence making the model calibration easier and 2) incorporate additional information to facilitate the prediction of the outer edge of a boundary layer corresponding of the so-called defect layer in a channel flow.

The modification is implemented into the "code-friendly" elliptic-blending based $\overline{v^2} - f$ model of Billard et al. (2008), namely the $\varphi - \alpha$ model, but could be used along with any ε based turbulence model. By resolving the elliptic blending parameter α and the near-wall anisotropy $\varphi = \overline{v^2}/k$ instead of $\overline{v^2}$ and f, the $\varphi - \alpha$ model was shown to address the numerical stability issue without impairing the predictive accuracy, unlike other "code-friendly" $\overline{v^2} - f$ formulations for which terms are neglected (Billard et al. (2008)). Moreover, since the primary concern of the $\varphi - \alpha$ development was the model robustness and its easy implementation into an industrial purpose segregated solver, the proposed ε equation modification follows the same philosophy. The resulting model has been validated using the open source finite volume collocated code *Code_Saturne* (Archambeau et al.). Validation results are presented in the cases of two pressure-induced separating flows, the asymmetric plane diffuser of Buice and Eaton (1997) and the flow over periodic hills of Temmerman and Leschziner (2001), where the correct prediction of the mixing layer between the bulk and the re-circulating flow is crucial.

1 Rationale

The ε equation in $\overline{v^2} - f$ models: The $\overline{v^2} - f$ models use the empirical form of the $k - \varepsilon$ equations as originally proposed by Jones and Launder (1972), reformulated as in Durbin (1991):

$$\begin{cases} \frac{Dk}{Dt} = P - \varepsilon + D_k^t + \nu \partial_j^2 k \\ \frac{D\varepsilon}{Dt} = \frac{1}{T}\left(C_{\varepsilon 1} P - C_{\varepsilon 2}\varepsilon\right) + D_\varepsilon^t + \nu \partial_j^2 \varepsilon \end{cases} \tag{1}$$

where $T = \max\left[k/\varepsilon, C_T\sqrt{\nu/\varepsilon}\right]$, $D_k^t = \partial_j\left(\nu_t/\sigma_k \partial_j k\right)$ and $D_\varepsilon^t = \partial_j\left(\nu_t/\sigma_\varepsilon \partial_j \varepsilon\right)$

This system determines baseline behaviour of fundamental flows, which involve only some of the source terms amongst P, ε, D_k^t and D_ε^t and this helps calibrate the constants associated to them, respectively $C_{\varepsilon 1}$, $C_{\varepsilon 2}$, σ_k and σ_ε.

Table 1 summarises the involvement of each term and constant in the different configurations: the coefficients $C_{\varepsilon 2}$ and $C_{\varepsilon 1}$ yield unique values of the decay rate in homogeneous isotropic turbulence (DIT) and turbulence growth rate in homogeneous shear turbulence (HST). Subsequently C_μ and σ_ε provide the desired value for $\frac{\overline{uv}}{k}$ and κ in the logarithmic region of a channel flow at infinite Reynolds number.

Mode	P	ε	D_k^t	D_ε^t	Constants
DIT		✓			$C_{\varepsilon 2}$
HST	✓	✓			$C_{\varepsilon 1}$
Defect Layer		✓	✓	✓	$C_{\varepsilon 2}$, σ_k, σ_ε
Log. Layer	✓	✓		✓	$C_{\varepsilon 1}$,$C_{\varepsilon 2}$, σ_ε
Near Wall	✓	✓	✓	✓	$C_{\varepsilon 1}$,$C_{\varepsilon 2}$,σ_k, σ_ε

Table 1: Role of the different k and ε source terms in the fundamental configurations

Adaptation to wall-bounded flows: Authors of $\overline{v^2} - f$ models have adapted this standard system to comply with wall-bounded flow requirements:

(a) A near-wall boosting of ε is needed in the buffer layer to represent production by local anisotropy (Durbin (1993))

(b) The constant $C_{\varepsilon 1}$ calibrated for homogeneous shear turbulence was shown to yield incorrect predictions in a boundary layer for which a larger value is recommended (Durbin (1995)).

To this end, the original model of Durbin (1991) (denoted in the following as DUR91) use a particularly large value of the coefficient $C_{\varepsilon 1}$. Durbin (1993) and subsequent authors suggested a functional $C_{\varepsilon 1}^*$ resulting in a blending between a larger near-wall value and the standard value calibrated in homogeneous flow. Durbin (1993) (DUR93) relies upon a $C_{\varepsilon 1}^*$ dependance on the production over dissipation ratio, whereas Durbin and Laurence (1996) (DUR96) alternatively uses $\sqrt{k/\overline{v^2}}$, noticing that the former suggestion would impair numerical stability. The same is used in Lien and Kalitzin (2001) (LIE01) (which is the $\overline{v^2} - f$ version used in main commercial codes, Fluent, Star-CD, StarCCM), in Manceau et al. (2002) and in Uribe (2006) (URI06). Following a similar reasoning as the Elliptic Blending Reynolds Stress Model (EBRSM) of Manceau and Hanjalić (2002) but applied in an eddy viscosity framework, the $\varphi - \alpha$ model of Billard et al. (2008)(BIL08) uses the same relation:

$$C_{\varepsilon 1}^* = 1.44\left(1 + 0.04(1-\alpha^3)\sqrt{k/\overline{v^2}}\right) \qquad (2)$$

where the elliptic blending coefficient α switches from 0 at the walls to 1 in the far field.

Two models introduce the wall-distance parameter: Lien and Durbin (1996) (LIE96) proposes a $C_{\varepsilon 1}^*$ dependancy on $R_y = y\sqrt{k}/\nu$ to achieve better prediction of by-pass transition, and Durbin (1995) relies on y to return distinct values of $C_{\varepsilon 1}^*$ in wall bounded and free shear flows. However this is in contradiction with the wall-distance free feature of $\overline{v^2} - f$ modelling and it is generally avoided by modellers.

Table 2 summarises the different terms and constants used for the ε equation and figure 1 shows *a posteriori* evaluation of $C_{\varepsilon 1}^*$ in a channel flow at $Re_\tau = 2000$.

For all models except DUR91 and DUR95, a very large value is returned in the near-wall region for the $C_{\varepsilon 1}^*$ coefficient. Note that neither DUR91 nor DUR95 feature ε production enhancement and therefore do not fulfil requirement (a).

Requirement (b) is satisfied for all models except BIL08. Apart from the latter model, $C_{\varepsilon 1}^*$ is always significantly larger than the standard value of 1.4-1.44. In BIL08, the use of the elliptic blending parameter α results in the model returning values of $C_{\varepsilon 1}^*$ in the channel flow considerably smaller than those of other models, and this has negative effects on wall-bounded flow predictions, as it will be seen later on.

Interdependence: For most of the models, the influence of the $C_{\varepsilon 1}^*$ coefficient modifications, proposed to meet requirements (a) and (b), extends beyond the zones for which they are intended for. This is visible in the channel flow, figure 1, where the $C_{\varepsilon 1}^*$ profiles show that for DUR96, LIE01, MAN02 and URI06 the use of the structural parameter $k/\overline{v^2}$ makes the $C_{\varepsilon 1}^*$ modification too intrusive: the influence of the near-wall boosting extends to a large part of the logarithmic region. The use of the wall-distance y in LIE96 and the elliptic blending parameter α in BIL08 enables to damp the $C_{\varepsilon 1}^*$ boosting outside the near-wall region.

Table 3 presents the predictions of the Von Kármán constant κ and the turbulent to mean strain time scale $\eta = S\frac{k}{\varepsilon}$ in the log region as well as the value taken by $\varphi = \overline{v^2}/k$. The last two columns give the values of η and $C_{\varepsilon 1}^*$ in HST for $St \rightarrow \infty$. Note that in that latter case, φ depends on whether the LRR-IP or the SSG model for pressure strain is used by correponding authors. The values are obtained by solving iteratively the corresponding simplified equations. As seen on table 3, the behaviour of the selected $\overline{v^2} - f$ models in the logarithmic layer noticeably varies from one another and κ often lies outside the range $[0.38 - 0.41]$ where the theoretical value should be. The first reason for that is the known non local "amplification" of the $\overline{v^2}$ redistribution term (Wizman et al. (1996), Manceau et al. (2001)), yielding a too large value of $\varphi_{log} = \overline{v^2}/k$ in the logarithmic layer for most of the models, with the worst effects in LIE96 and LIE01. The influence of this adverse effect on prediction of κ is all the more important for models for which $C_{\varepsilon 1}^*$ depends on φ. However solutions to the amplification effect were proposed by the same authors and the problem was addressed in DUR96, MAN02 and BIL08. The switch from elliptic relaxation to elliptic blending in the latter model guarantees no amplification. Secondly, the too large influence of the near-wall region $C_{\varepsilon 1}^*$ modification has led modellers to adopt various standard values of coefficients $C_{\varepsilon 1}$, $C_{\varepsilon 2}$ and σ_ε, since this inter-dependance makes all these coefficients function of $C_{\varepsilon 1}^*$, but these readjustments are not enough to enable the theoretical Von Kármán constant κ to be recovered.

It is also worth mentioning that for all reviewed models except DUR95 and BIL08 the coefficient $C_{\varepsilon 1}^*$ never returns its standard value calibrated for homogeneous shear turbulence. As seen in table 3, for all these models the value of this coefficient, $C_{\varepsilon 1,\infty}^*$ is of the same order as the value adopted in a channel flow,

Model	$C^*_{\varepsilon 1}$	$C_{\varepsilon 2}$	σ_ε	C_μ
DUR91	1.7	2.0	1.3	0.2
DUR93	$1.44\left(1+0.1\frac{P}{\varepsilon}\right)$	1.9	1.3	0.23
DUR95	$1.3 + 0.25/\left[1+\left(\frac{C_L y}{2L}\right)^8\right]$	1.9	1.3	0.19
DUR96	$1.44\left(1+\frac{1}{30}\sqrt{\frac{k}{v^2}}\right)$	1.85	1.5	0.16
LIE96	$1.55 + exp\left(-0.00285R_y^2\right)$	1.92	1.5	0.19
LIE01	$1.4\left(1+0.05\sqrt{\frac{k}{v^2}}\right)$	1.9	1.3	0.22
MAN02	$1.44\left(1+0.06\sqrt{\frac{k}{v^2}}\right)$	1.91	1.3	0.22
URI06	$1.4\left(1+0.05\sqrt{\frac{k}{v^2}}\right)$	1.85	1.3	0.22
BIL08	Eq.2	1.83	1.22	0.22

Table 2: Values of the ε coefficients for the different models

Model	κ	η_{log}	φ_{log}	η_∞	$C^*_{\varepsilon 1,\infty}$
DUR91	0.34	3.31	0.46	4.55	1.70
DUR93	0.37	3.01	0.48	4.43	1.58
DUR95	0.41	2.72	0.71	6.78	1.30
DUR96	0.36	3.84	0.42	5.08	1.52
LIE96	0.51	2.16	1.13	4.87	1.55
LIE01	0.59	1.69	1.60	4.64	1.52
MAN02	0.40	3.20	0.44	4.43	1.53
URI06	0.41	2.77	0.59	4.49	1.51
BIL08	0.38	3.23	0.44	4.59	1.44

Table 3: Behaviour of the models in the logarithmic region and in homogeneous shear turbulence

in contrast to requirement (b). The model of DUR95 is the only one able to recover the smaller value of 1.3 in free shear flows albeit by using the wall-distance.

2 The present proposal

The use of α in BIL08 enables the requirement (a) to be met and the model behaviour outside the near-wall region, as well as in the homogeneous flows is not affected by the $C^*_{\varepsilon 1}$ modification. But requirement (b) is not satisfied because a smaller value of 1.44 is returned right from the inner edge of the logarithmic layer of a channel flow, this being an adverse consequence of the presence of α in the $C^*_{\varepsilon 1}$ definition. Whereas Durbin (1995) uses the wall-distance to characterise a wall bounded flow, the present work revisits an idea originally formulated in Parneix et al. (1996) who proposed to use information about the turbulent transport of turbulence to characterise the edge of a boundary layer, corresponding to the defect layer of a channel flow. Indeed, analysis of DNS data in channel flow shows

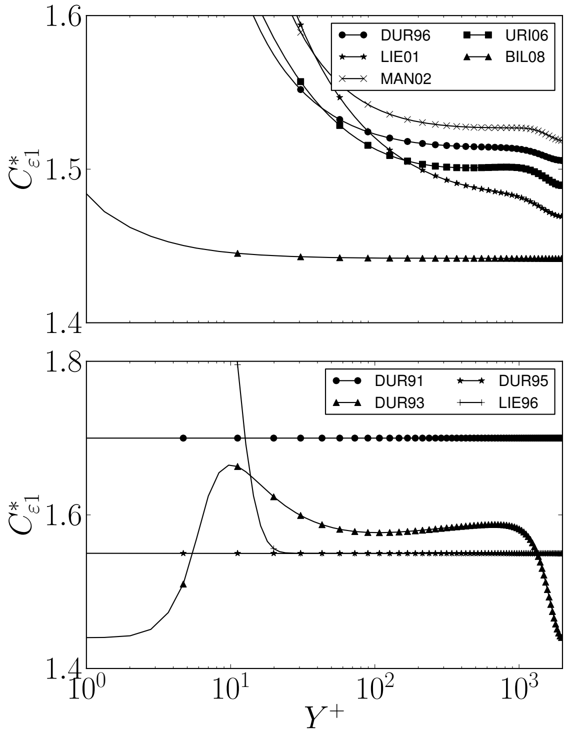

Figure 1: $C^*_{\varepsilon 1}$ in a channel flow for $Re_\tau = 2000$. Top: Models for which $C^*_{\varepsilon 1}$ depends on $\frac{\overline{v^2}}{k}$. Bottom: Other models

that above the logarithmic layer, as velocity gradient decreases, so does P, the turbulence is sustained by transport terms, gradually becoming more and more important towards the defect layer. In the k equation, the equilibrium $P = \varepsilon$ is then replaced by $D^t_k = \varepsilon$ towards the centre of the channel, as represented in table 1. The standard values of the ε equation constants are calibrated in a channel flow to represent the logarithmic layer only. An analysis of the budget of the ε equation, as performed in Parneix et al. (1996) shows that the exact ε source term $P_1+P_2+P_3+P_4-Y$ (using the same notation as Mansour and Rodi (1993)) is loosely represented by the standard values $C_{\varepsilon 1} = 1.44$ and $C_{\varepsilon 2} = 1.83$ from the upper edge of the logarithmic layer, and Parneix et al. (1996) recommends $C_{\varepsilon 2}$ to be halved in the defect layer. To this end, the latter authors suggest a $C_{\varepsilon 2}$ dependancy on D^t_ε and P. Noteworthily, a better representation of the defect layer cannot be achieved by a modification of $C^*_{\varepsilon 1}$ since the production is zero in this region. Following the same idea, a functional $C^*_{\varepsilon 2}$ is proposed for the $\varphi-\alpha$ model:

$$C^*_{\varepsilon 2} = C_{\varepsilon 2} + \alpha^p\left(C_{\varepsilon 4} - C_{\varepsilon 2}\right)\tanh\left(\left|\frac{D^t_k}{\varepsilon}\right|^{3/2}\right) \quad (3)$$

This results in $C^*_{\varepsilon 2}$ going from the standard value $C_{\varepsilon 2}$ in the logarithmic region to a decreased value of $C_{\varepsilon 4}$ in the defect layer. The $\varphi - \alpha$ model integrating this $C^*_{\varepsilon 2}$ modification is described by equations 4, 5 and 6 and the values adopted by the constants are given in table 4. Only coefficients C_L and C_η needed slight readjustment. The inclusion of the blending parameter α in relation 3 ensures the $C^*_{\varepsilon 2}$ modification is not active near the wall, where turbulent transport is present however.

$$\begin{cases} \frac{Dk}{Dt} = P - \varepsilon + \partial_j \left(\left(\nu + \frac{\nu_t}{\sigma_k} \right) \partial_j k \right) \\ \frac{D\varepsilon}{Dt} = \frac{1}{T} \left(C^*_{\varepsilon 1} P - C^*_{\varepsilon 2} \varepsilon \right) + \partial_j \left(\left(\nu + \frac{\nu_t}{\sigma_\varepsilon} \right) \partial_j \varepsilon \right) \end{cases} \tag{4}$$

$$\begin{cases} \lim\limits_{y \to 0} k = 0 \\ \lim\limits_{y \to 0} \varepsilon = \lim\limits_{y \to 0} \dfrac{2\nu k}{y^2} \end{cases} \quad \begin{cases} L^2 \Delta\alpha - \alpha = -1 \\ \lim\limits_{y \to 0} \alpha = 0 \end{cases} \tag{5}$$

$$\begin{cases} \frac{D\varphi}{Dt} &= (1-\alpha^p) \left[-\varphi \frac{\varepsilon}{k} \right] + \alpha^p f_h \\ & \quad -P \frac{\varphi}{k} + \frac{2}{k} \left(\frac{\nu_t}{\sigma_k} + \alpha^p \nu \right) \partial_j k \partial_j \varphi \\ & \quad +\partial_j \left(\left(\nu + \frac{\nu_t}{\sigma_\varphi} \right) \partial_j \varphi \right) \\ f_h &= -\frac{1}{T} \left(C_1 - 1 + C_2 \frac{P}{\varepsilon} \right) \left(\varphi - \frac{2}{3} \right) \\ \lim\limits_{y \to 0} \varphi &= 0 \end{cases} \tag{6}$$

C_T	C_L	C_η	C_1	C_2	σ_φ	p
6	0.164	86	1.7	1.2	1	3
$C^*_{\varepsilon 1}$	$C^*_{\varepsilon 2}$	σ_k	σ_ε	ν_t		C_μ
Eq.2	Eq.3	1	1.22	$C_\mu \varphi k T$		0.22
T			L			
$\max\left[\frac{k}{\varepsilon}, C_T \sqrt{\frac{\nu}{\varepsilon}}\right]$			$C_L \max\left[\frac{k^{3/2}}{\varepsilon}, C_\eta \left(\frac{\nu^3}{\varepsilon}\right)^{1/4}\right]$			

Table 4: Constants of the present model.

The *a priori* evaluation of $C^*_{\varepsilon 2}$ given by relation 3 for different Reynolds numbers in a channel flow is shown in figure 2. This relation yields a fairly Re_τ independent characterisation of the central region of the channel. As achieved in Parneix et al. (1996), the exact source term of the ε equation is better represented using the proposed $C^*_{\varepsilon 2}$, as shown in figure 3, whereas the standard value 1.83 yields a too strongly negative ε sink term. This discrepancy of the standard model returns insufficient level of dissipation in this region resulting in a well-known over-estimation of the turbulent viscosity (Laurence et al. (2004)).

The predicted velocity and turbulent viscosity given by the BIL08 model and the present proposal are shown in figure 4 for the channel flow case at $Re_\tau = 2000$. The turbulent viscosity returned by the model BIL08 is over-predicted in the central region, and this discrepancy is common to all $\overline{v^2} - f$ models, they lack information characterising the defect layer.

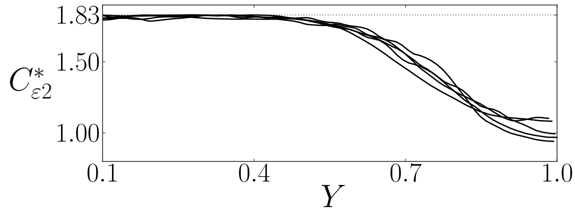

Figure 2: *A priori* evaluation of $C^*_{\varepsilon 2}$ from Eq.3 for $Re_\tau \in \{180; 395; 590; 950; 2000\}$

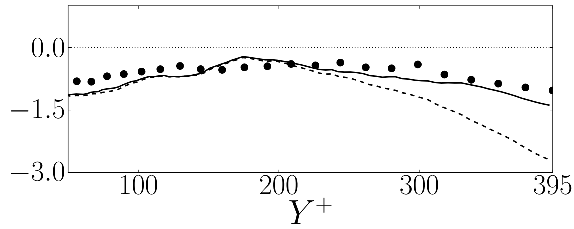

Figure 3: Main source term of ε, $P_1 + P_2 + P_4 - Y$, in the central region of a channel flow, $Re_\tau = 395$. ●●●: Exact term (from Mansour and Rodi (1993)). Model $\left(1.44P - C^*_{\varepsilon 2}\varepsilon\right)/T$ with $C^*_{\varepsilon 2} = 1.83$ (— — —) and with $C^*_{\varepsilon 2}$ from Eq.3 (——) (all terms $\times \left(y^+\right)^2$)

This is to be directly linked to the consistent ε under-prediction

This ν_t over-prediction is however moderate with BIL08 but was shown to be considerably larger for models over-predicting $\overline{v^2}$ (for those depicting a strong "amplification" effect, such as the popular "code-friendly version of LIE96 and LIE01, as shown in Uribe (2006)).

The present version yields a improved representation of ν_t in the central region, compared to BIL08, thus leading to a improved mean velocity prediction. The two models behave very similarly elsewhere.

With this $C^*_{\varepsilon 2}$ coefficient modification the $\varphi - \alpha$ model satisfies both requirement (a) and (b). The spreading rate control achieved by other $\overline{v^2} - f$ models using a higher value for $C^*_{\varepsilon 1}$ in wall bounded flows is now ensured by a gradually decrease of $C^*_{\varepsilon 2}$ from the outer edge of the logarithmic layer. Recalling that the turbulence growth P/ε in a mixing layer is proportionnal to $\left(C_{\varepsilon 2} - 1\right) / \left(C_{\varepsilon 1} - 1\right)$ then a increase of $C_{\varepsilon 1}$ is equivalent to a reduction of $C_{\varepsilon 2}$.

3 Results on the pressure-induced separating flows

The proposed modification was tested on two wall-bounded separating flows: the asymmetric plane diffuser of Buice and Eaton (1997) and the flow over periodic hills of Temmerman and Leschziner (2001). Because of the flows complexity, the three dimensional

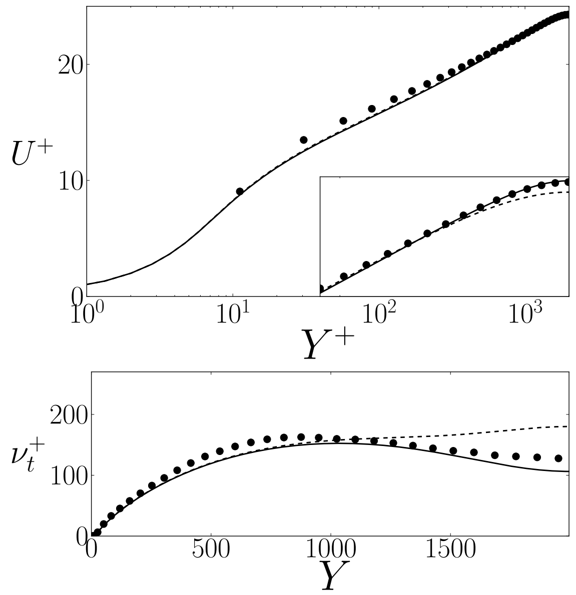

Figure 4: Prediction of the velocity profile (top) and the turbulent viscosity (bottom) in a channel flow, $Re_\tau = 2000$. • • •: DNS, ━━ : Present model, — — — : BIL08

and transient nature of the separation, the relevance of simple eddy viscosity modelling in these cases may be questioned (e.g. Sveningsson et al. (2005)), but they have often been used to assess $\overline{v^2} - f$ capabilities and $C^*_{\varepsilon 1}$ modifications (Durbin and Laurence (1996), Manceau et al. (2002), Iaccarino (2001)).

Figure 5 compares the periodic hill flow streamlines predicted by the $\varphi - \alpha$ without and with the $C^*_{\varepsilon 2}$ modification, the $\overline{v^2} - f$ model of LIE01, the $k - \omega$ SST model of Menter (1994) and the reference LES calculation. The model LIE01 represents a widely used and validated $\overline{v^2} - f$ version because it is the one adopted by various CFD codes and is acknowledged to yield very good predictions in such flows (Iaccarino (2001)). On the other hand, the $k - \omega$ SST model strongly over-predicts the re-circulation extent.

The model of BIL08 severely underestimates the re-circulating flow. This is directly linked to an over-prediction of the turbulent shear stress throughout the domain because of the too small value of $C^*_{\varepsilon 1}$ returned in wall bounded flow, and the $C^*_{\varepsilon 2}$ modification is intended to remedy this issue. The present proposal indeed returns larger re-circulation compared to BIL08, now of the same order as LIE01.

The same conclusion holds for the diffuser flow: figure 6 represents the prediction of the re-circulation for the same models, in terms of skin-friction and pressure coefficient. As predicted by BIL08 the flow does not separate whereas the present modification enables the $\varphi - \alpha$ model to yield a separation and reattachement location close to the one observed experimentally and the predicted re-circulation is larger, yielding a smaller pressure coefficient.

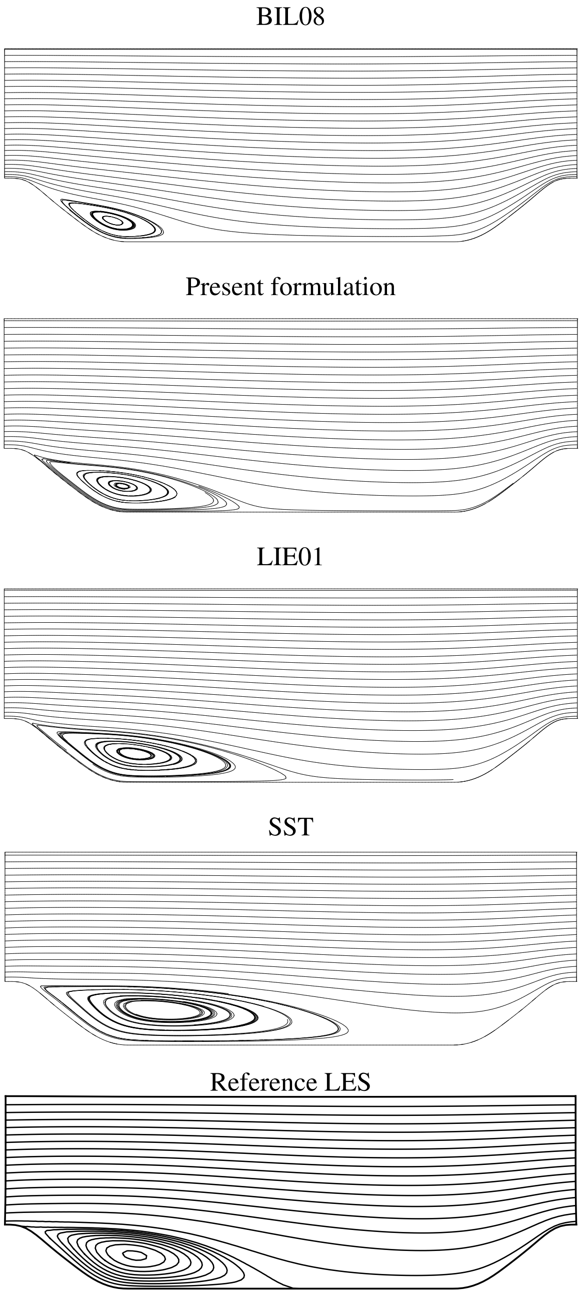

Figure 5: Streamlines of the periodic hill flow. From top to bottom: BIL08, present model, LIE01, $k - \omega$ SST and LES of Temmerman and Leschziner (2001)

4 Conclusion

The present $\varphi - \alpha$ improvement enables the model to perform as well as the "standard" $\overline{v^2} - f$ of Lien and Kalitzin (2001) in wall bounded flows with a modification intended to leave unchanged the behaviour in other configurations. Therefore the superior behaviour of the $\varphi - \alpha$ in buoyancy driven and relaminarizing flows (Billard et al. (2008)) is maintained. In the improved version, more information is provided to the

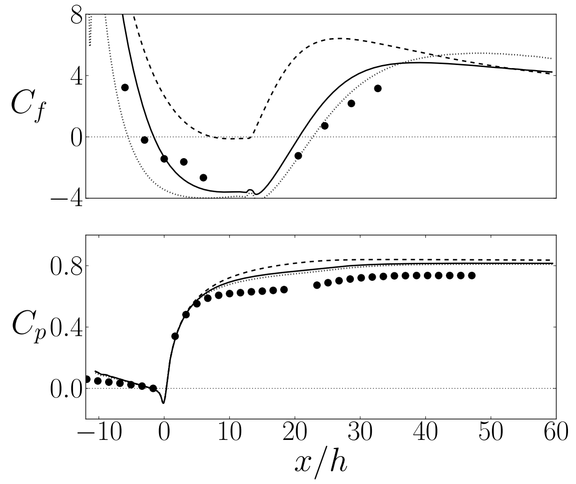

Figure 6: Friction coefficient $\times 10^3$ (top) and pressure coefficient (bottom) in the diffuser case: • • •: Experiment, ▬▬ : Present model, — — — : BIL08, · · · : LIE01

dissipation rate transport equation, taking the form of two parameters: The blending coefficient α takes the value 0 in a thin near-wall layer and 1 elsewhere. The turbulent transport of k over ε ratio (D_k^t/ε) takes the value 1 at the edge of a boundary layer and 0 elsewhere. The combination of both in the functionnal coefficients $C_{\varepsilon 1}^*$ and $C_{\varepsilon 2}^*$ is a way to correctly reproduce different flow configurations in an independent way. The presence of α in the $C_{\varepsilon 1}^*$ definition helps calibrate the near-wall behaviour of the turbulent scales without affecting other parts of the flow, and the coefficient $C_{\varepsilon 4}$ included in the $C_{\varepsilon 2}^*$ definition can be modified to achieve better predictions of wall-bounded flows without changing the model behaviour in homogeneous flows.

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