Simulations of flows over spill weirs with *Code_Saturne*

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- Project initiated by EDF R&D studying geometries of spill weir

  Goal: enhance adherence, limit low pressures on the weir and delay dynamic flow separation up to higher upstream water heights.
- Volume Of Fluid method new in *Code_Saturne* V5.0.

Outlines

1. VOF model
2. Thin weir
3. Creager weir
4. Piano Key (PK) Weir
5. References

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VOF model

Starting with Navier-Stokes equation:

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \underline{\nabla}.(\rho \underline{u}) = 0 \\ \dfrac{\partial \rho \underline{u}}{\partial t} + \underline{\nabla}.(\rho \underline{u} \otimes \underline{u} - \underline{\underline{T}}) = \rho \underline{g} \end{cases} \tag{1}$$

with $\underline{\underline{\mathrm{T}}}$ the stress tensor:

$$\underline{\underline{T}} = -(P + \frac{2}{3}\mu \underline{\nabla}.\underline{u})\underline{\underline{1}} + \mu(\underline{\underline{\nabla}}\ \underline{u} + (\underline{\underline{\nabla}}\ \underline{u})^T)$$

VOF model

- An air volume fraction (often called void fraction) is defined: α.
- Mixing laws are used:

$$\rho = \alpha\rho_{air} + (1-\alpha)\rho_{water} \tag{2}$$

$$\mu = \alpha\mu_{air} + (1-\alpha)\mu_{water} \tag{3}$$

- By injecting (2) in the mass balance equation, we get:

$$\frac{\partial(\alpha\rho_{air} + (1-\alpha)\rho_{water})}{\partial t} + \underline{\nabla}.\left([\alpha\rho_{air} + (1-\alpha)\rho_{water}]\underline{u}\right) = 0$$

$$\Leftrightarrow (\rho_{air} - \rho_{water})[\frac{\partial\alpha}{\partial t} + \underline{\nabla}.(\alpha\underline{u})] + \rho_{water}\underline{\nabla}.(\underline{u}) = 0$$

- and the following equivalence between the transport equation of the void fraction and the incompressibility equation:

$$\boxed{\frac{\partial\alpha}{\partial t} + \underline{\nabla}.(\alpha\underline{u}) = 0 \;\Leftrightarrow\; \underline{\nabla}.(\underline{u}) = 0} \tag{4}$$

VOF model

VOF model:

$$\underline{\nabla}.(\underline{u}) = 0$$

$$\frac{\partial \rho \underline{u}}{\partial t} + \underline{\nabla}.(\rho \underline{u} \otimes \underline{u}) = \rho \underline{g} - \underline{\nabla} P + \underline{\nabla}.(\mu(\underline{\underline{\nabla}}\ \underline{u} + (\underline{\underline{\nabla}}\ \underline{u})^T - (\frac{2}{3}\mu \underline{\nabla}.\underline{u})\underline{\underline{1}})$$

$$\frac{\partial \alpha}{\partial t} + \underline{\nabla}.(\alpha \underline{u}) = 0$$

Momentum balance under its solved form, with $P^* = P - \rho_{air}\underline{g}$ the solved pressure:

$$\rho \frac{\partial \underline{u}}{\partial t} + \underline{\nabla}.(\rho \underline{u} \otimes \underline{u}) - \underline{u}\ \underline{\nabla}.(\rho \underline{u})$$
$$= (\rho - \rho_{air})\underline{g} - \underline{\nabla} P^* + \underline{\nabla}.(\mu(\underline{\underline{\nabla}}\ \underline{u} + (\underline{\underline{\nabla}}\ \underline{u})^T) - (\frac{2}{3}\mu \underline{\nabla}.\underline{u})\underline{\underline{1}})$$

VOF algorithm in *Code_Saturne*

- Prediction step: computation of $\underline{\tilde{u}}$

$$\rho\frac{\underline{\tilde{u}}-\underline{u}^n}{\Delta t}+\underline{\nabla}.((\rho\underline{u})^n\otimes\underline{\tilde{u}})-\underline{\tilde{u}}\,\underline{\nabla}.((\rho\underline{u})^n)=(\rho-\rho_{air})\underline{g}-\underline{\nabla}P^n+\underline{\nabla}.(\mu(\underline{\underline{\nabla}}\,\underline{\tilde{u}}+(\underline{\underline{\nabla}}\,\underline{\tilde{u}})^T-(\frac{2}{3}\mu\underline{\nabla}.\underline{\tilde{u}})\underline{\underline{1}})$$

- Correction step:

$$\delta P=P^{n+1}-P^n$$

and:

$$\begin{cases}\dfrac{\underline{u}^{n+1}-\underline{\tilde{u}}}{\Delta t}=-\dfrac{1}{\rho}\underline{\nabla}(\delta P)\\ \underline{\nabla}.\underline{u}^{n+1}=0\end{cases}$$

yielding:

$$\underline{\nabla}.(\underline{\tilde{u}})=\underline{\nabla}.(\frac{\Delta t}{\rho}\underline{\nabla}(\delta P))$$

δP_{cell} then $(\underline{u}^{n+1}.\underline{S})_{f_{ij}}$ are computed, and finally $\underline{u}^n_{cell}$ is updated to $\underline{u}^{n+1}_{cell}$

- Resolution of the void fraction transport equation and update of $(\rho\underline{u}.\underline{S})_f$ with an upwind scheme and of mixing properties ρ and μ

$$\frac{(\alpha_l^{n+1}-\alpha_l^n)}{\Delta t}+\underline{\nabla}.(\alpha\underline{u})^{n+1}=0$$

- Possibility to iterate over these steps to make $\underline{\tilde{u}}$ converge towards $\underline{u}^{n+1}$ (in remaining terms appearing in the sum of prediction and correction equations).

VOF model

Discretization over the cell Ω_i and between t_n and t_{n+1}:

$$\frac{\partial \alpha}{\partial t} \rightarrow \frac{|\Omega_i|}{\Delta t}(\alpha_I^{n+1} - \alpha_I^n)$$

$$\underline{\nabla}.(\alpha \underline{u}) \rightarrow \sum_f [(1-\theta)(\alpha_f \underline{S}_f.\underline{u}_f)^n + \theta(\alpha_f \underline{S}_f.\underline{u}_f)^{n+1}]$$

where:

- f face of Ω_i
- $\underline{S}_f$ surface vector of face f
- α_f face value of α (interpolated)
- $\underline{u}_f$ face value of $\underline{u}$ (interpolated)

Discrete transport equation applying an implicit Euler time scheme ($\theta = 1$):

$$\boxed{\frac{|\Omega_i|}{\Delta t}(\alpha_I^{n+1} - \alpha_I^n) + \sum_f (\alpha_f \underline{S}_f.\underline{u}_f)^{n+1} = 0}$$

VOF model

Numerical scheme choice

Necessary properties:

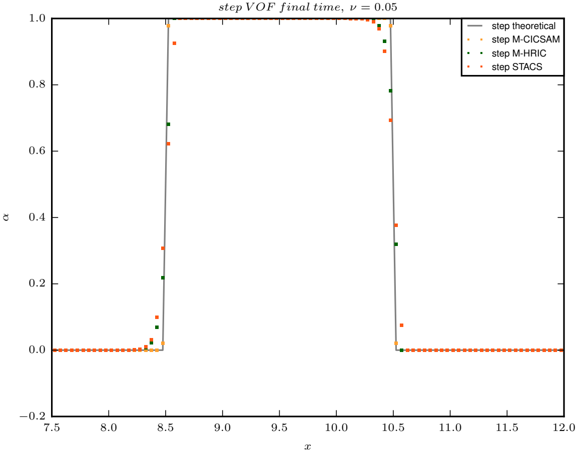
- α bounded between 0 and 1 - min/max principle.
- avoid interface diffusion.

Numerical convection schemes implemented in *Code_Saturne* for VOF computation: STACS [2], M-HRIC [4], M-CICSAM [7]

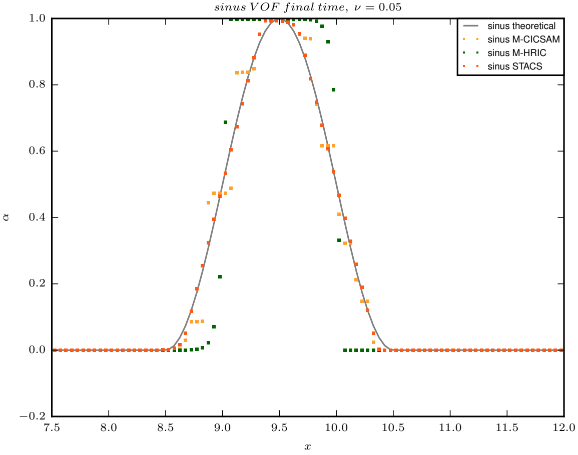
Principle: blend a very compressive scheme and a Total Variation Diminushing scheme, with a weighting factor depending on angle between the free surface and the face normal, and on the Courant number Co.

VOF model

Convection scheme comparisons - 1D convection of a step signal



Convection scheme comparisons - 1D convection of a sinus signal



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Simulation of a flow over a thin weir

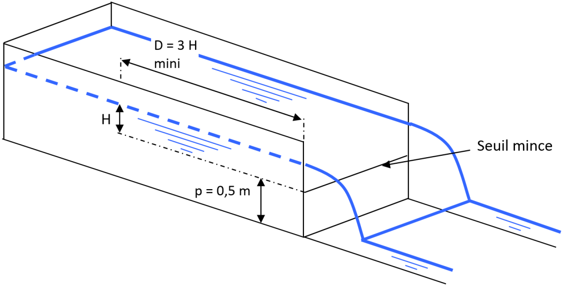
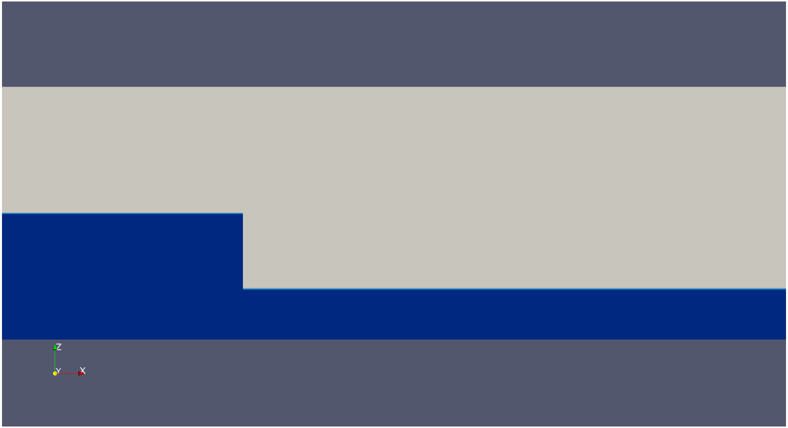


Figure: Sketch of a spilling way with a thin weir

Thin weir



Thin weir

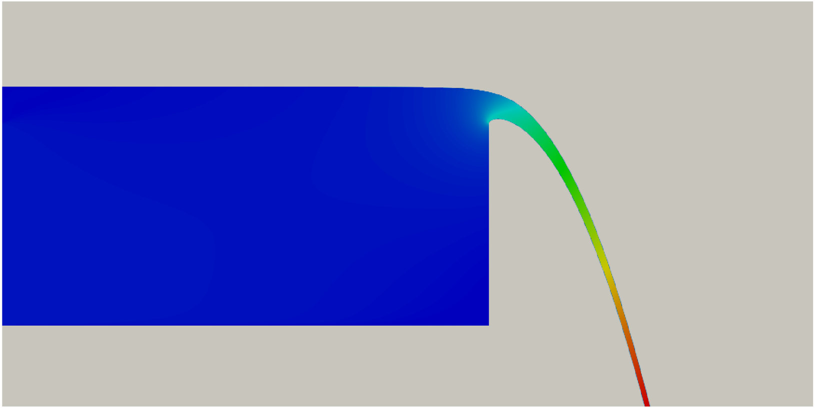


Figure: Velocity for $q = 0.1 m^2/s$

Thin weir

Validation of *Code_Saturne* simulations by comparison with empirical laws:

- Height-Flow rate law:

$$Q = C_D b\sqrt{2g}H^{3/2}$$

$$C_D = 0.4023(\frac{h_e}{H})^{3/2}(1 + 0.135\frac{h_e}{p}) \quad (Rehbock\ [3])$$

$$with\ h_e = H + 0.0011$$

- Profile of the lowest free surface of the water flow:

$$\frac{z}{H} = \frac{1}{2}(\frac{x}{H})^{1.85} \quad (Scimemi\ [5])$$

$$\frac{z}{H} = 0.556(\frac{x}{H})^2 \quad (De\ Marchi\ [5])$$

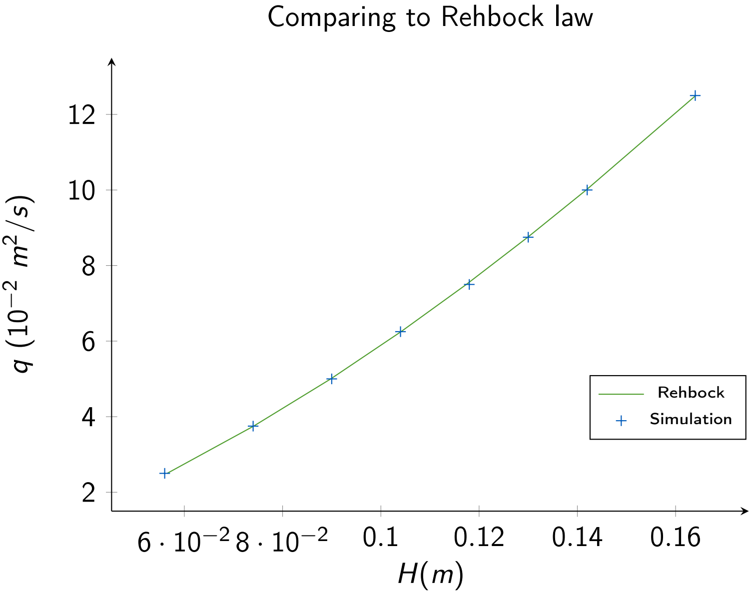
Thin weir - comparison to Height-Flow rate laws

Free surface defined for the criterium $\alpha = 0.5$, for the 2nd mesh, the thickness of the free surface ($\alpha \in [0.001, 0.999]$) is not bigger than one cell height (2mm) for all flow rates.

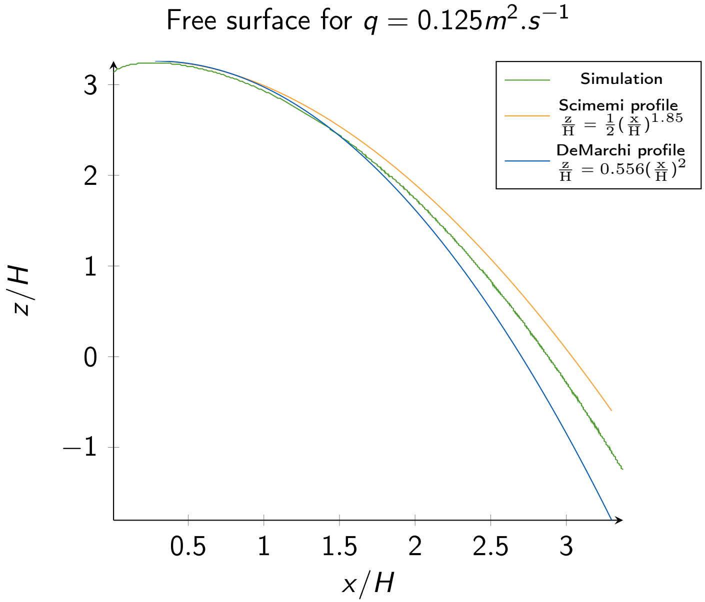
Débit (m^2/s)	H_{th} (m)	H_{simu} (m) $\pm 2mm$	H_{simu} (m) $\pm 1mm$	Écart %	C_D Rehbock
0.025	0.0565	0.056	0.056	-0.85	0.4205
0.0375	0.0741	0.0726	0.074	-0.15	0.4196
0.05	0.0898	0.088	0.09	0.27	0.4198
0.0625	0.1041	0.104	0.104	-0.06	0.4203
0.075	0.1174	0.116	0.118	0.52	0.4210
0.0875	0.1299		0.13	0.06	0.4218
0.1	0.1418	0.14	0.142	0.13	0.4227
0.125	0.1641	0.162	0.164	-0.06	0.4245

Table: Values of the height H for several flow rates

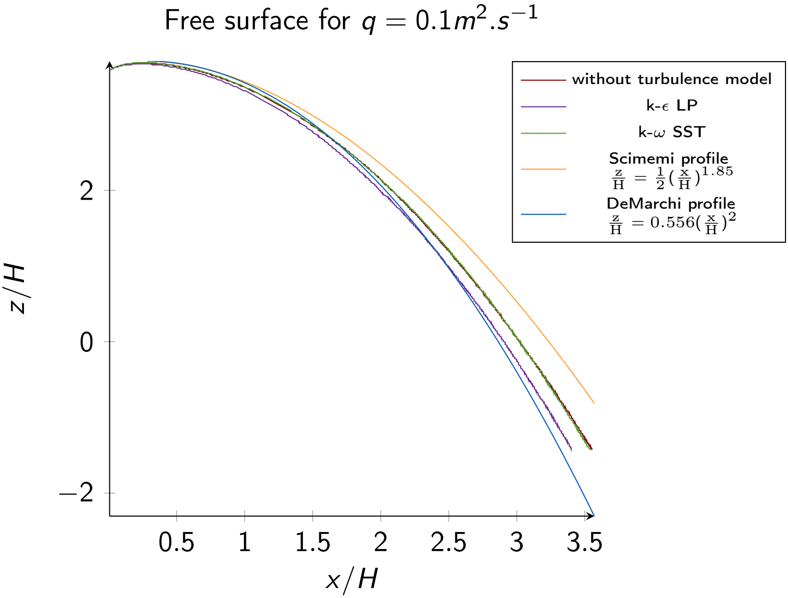
Thin weir - comparison to Height-Flow rate laws



Thin weir - lowest free surface profile



Thin weir - Turbulence model sensitivity



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Simulation of a flow spilling over a Creager weir
chosen (dimensioning) height: 20 cm (reduced size)

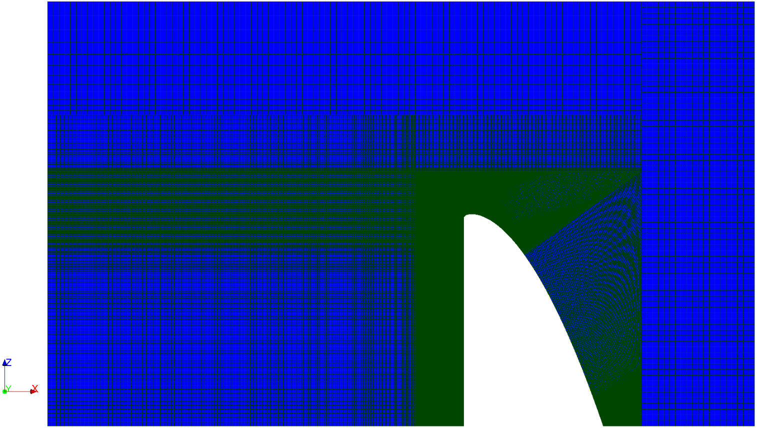


Figure: Mesh of the Creager weir

Creager weir

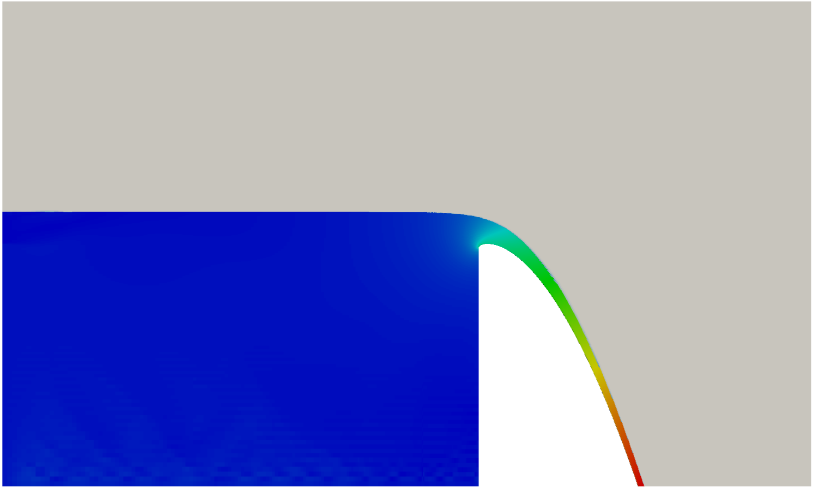


Figure: velocity field for $q = 0.2m^2/s$

Validation of the simulations by comparing with empirical laws:

- Height-flow rate:

$$Q = C_D b \sqrt{2g} H^{3/2}$$

$$C_D = 0.4950\left(\frac{H}{H_D}\right)^{0.12} \quad \textit{pour}\ (0.2 < \frac{H}{H_D} < 2) \quad (\textit{Brudenell}\ [6])$$

- Free surface profile (Vischer and Hager [1]):

$$S = 0.75(\chi^{1.1} - \frac{1}{6}X) \quad \textit{for}\ (-2 < X/\chi^{1.1} < 2)$$

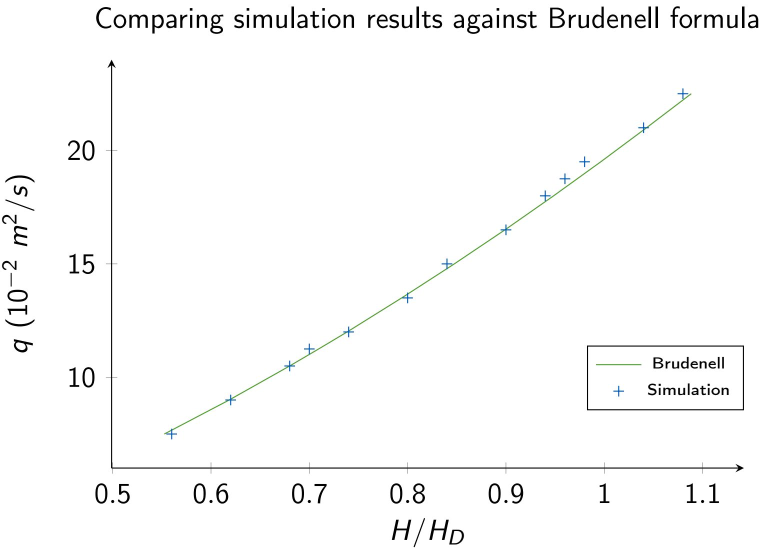
$$\textit{with}\ S = s/H_D \quad \chi = H/H_D \quad X = x/H_D$$

Creager weir - Height-Flow rate law

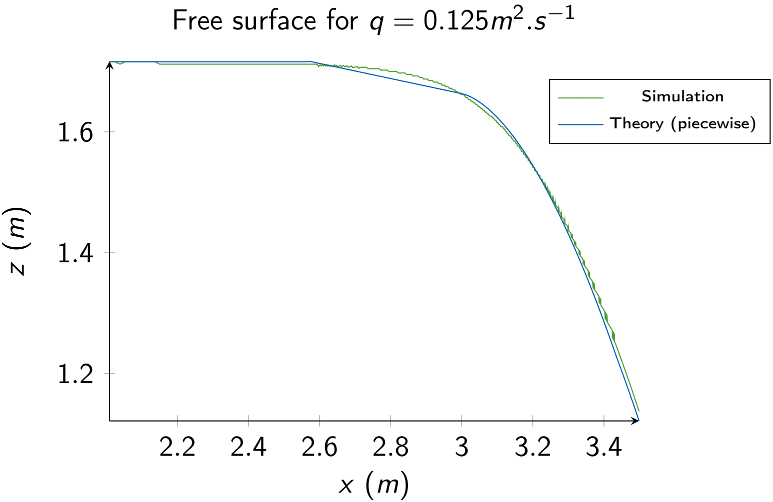
Flow rate (m^2/s)	H_{th} (m)	H_{simu} (m) $\pm 2mm$	Deviation %	C_D	H/H_D
0.075	0.1105	0.112	1.36	0.461	0.55
0.09	0.1237	0.124	0.27	0.467	0.62
0.105	0.1360	0.136	-0.01	0.473	0.68
0.1125	0.1419	0.14	-1.36	0.475	0.71
0.12	0.1477	0.148	0.21	0.477	0.74
0.135	0.1588	0.16	0.74	0.482	0.79
0.15	0.1695	0.168	-0.88	0.485	0.85
0.165	0.1798	0.18	0.13	0.489	0.90
0.18	0.1897	0.188	-0.89	0.492	0.95
0.1875	0.1945	0.192	-1.30	0.493	0.97
0.195	0.1993	0.196	-1.65	0.495	1.00
0.21	0.2086	0.208	-0.30	0.498	1.04
0.225	0.2177	0.216	-0.78	0.500	1.09
0.525	0.3673	0.36	-1.98	0.533	1.84
0.6	0.3989	0.39	-2.22	0.538	1.99

Table: Values of the height H - Surface: $\alpha = 0.5$ - $H_D = 0.2m$

Creager weir - Height-Flow rate law



Creager weir - free surface profile



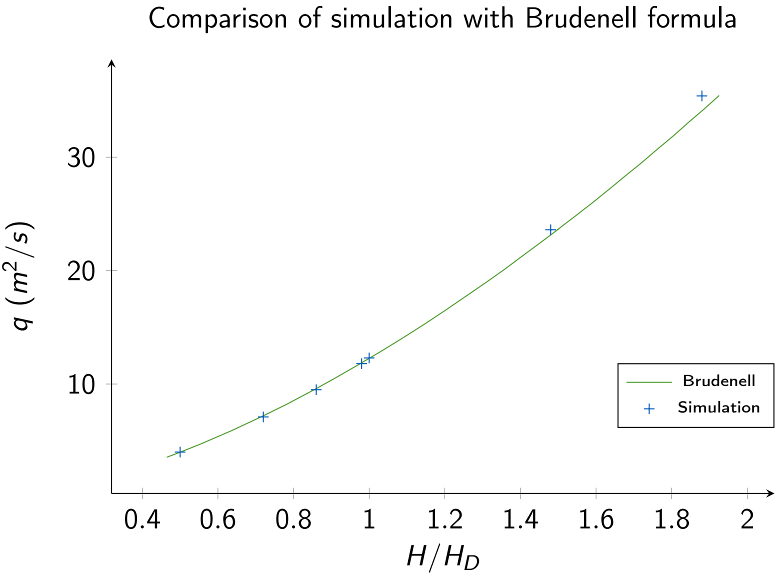
Real size: $H_D = 3.15m$ (for the comparison with *Flow3D* results on a validation case used by hydraulic engineers at CIH)

Uncertainty on height measurement: $\pm 0.063m$

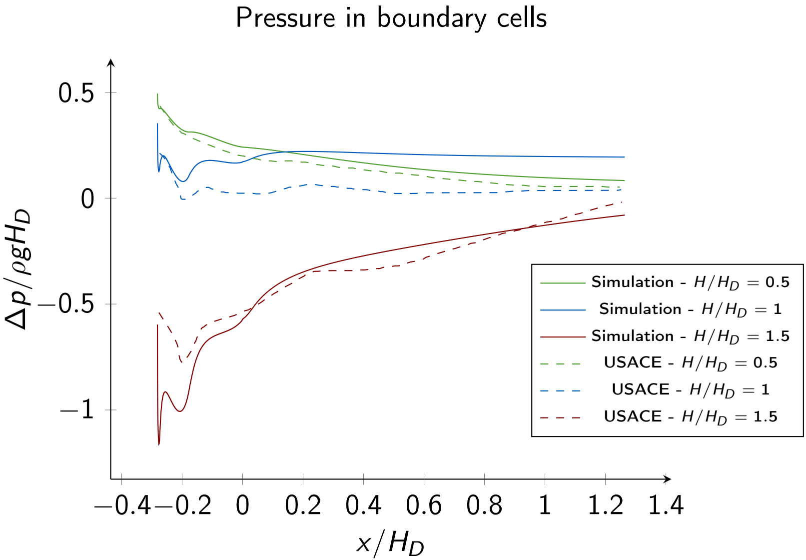
Flow rate (m^2/s)	H_{th} (m)	H_{simu} (m)	Deviation %	H/H_D	$C_{D\ th}$	$C_{D\ simu}$	Deviation %
4.0	1.576	1.575	-0.1	0.50	0.4555	0.4555	-0.01
7.1	2.246	2.268	1.0	0.71	0.4753	0.4759	0.12
9.5	2.683	2.709	1.0	0.85	0.4856	0.4861	0.12
11.8	3.079	3.087	0.3	0.98	0.4936	0.4938	0.03
12.3	3.154	3.15	-0.1	1.00	0.4951	0.4950	-0.02
23.6	4.723	4.662	-1.3	1.50	0.5197	0.5188	-0.16
35.4	6.066	5.922	-2.4	1.93	0.5355	0.5340	-0.29

Table: Values of height H for several flow rates - $H_D = 3.15m$

Large scale Creager - comparison with height-flow rate law



Large scale Creager - pressure along surface of the weir



Large scale Creager - reduced Creager and flow separation

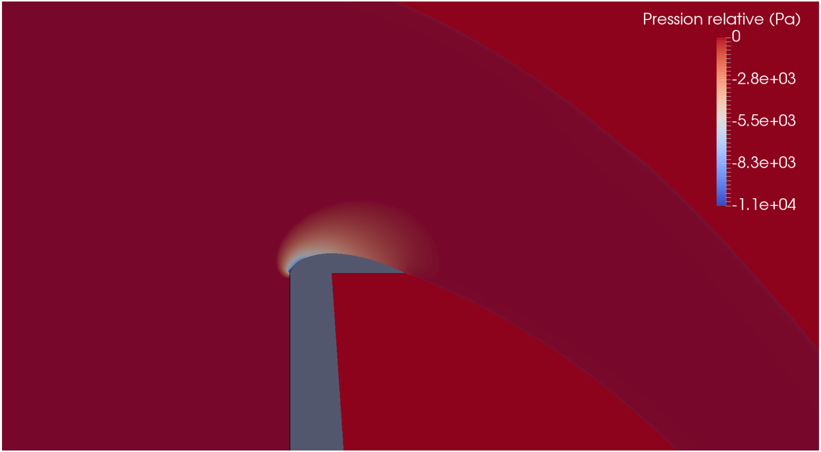


Figure: Pressure along reduced Creager weir for $H/H_D = 2.5$

Creager weir - 3D case

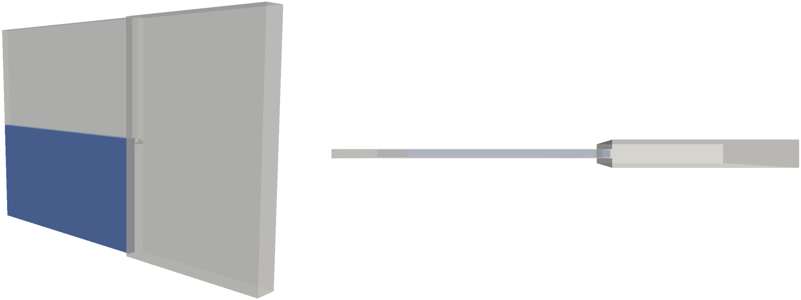


Figure: Geometry for the Creager weir 3D simulation - Initialisation

Creager weir - 3D case

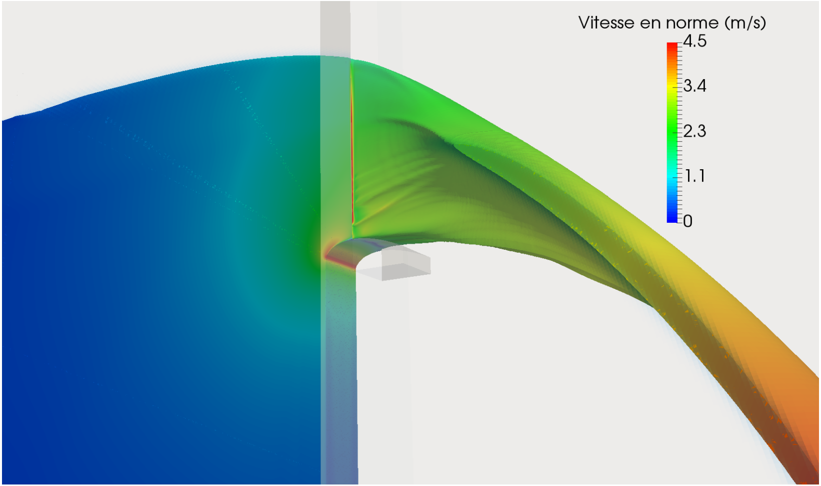


Figure: Flow separating from the weir for $H/H_D = 2.3$

Creager weir - 3D case

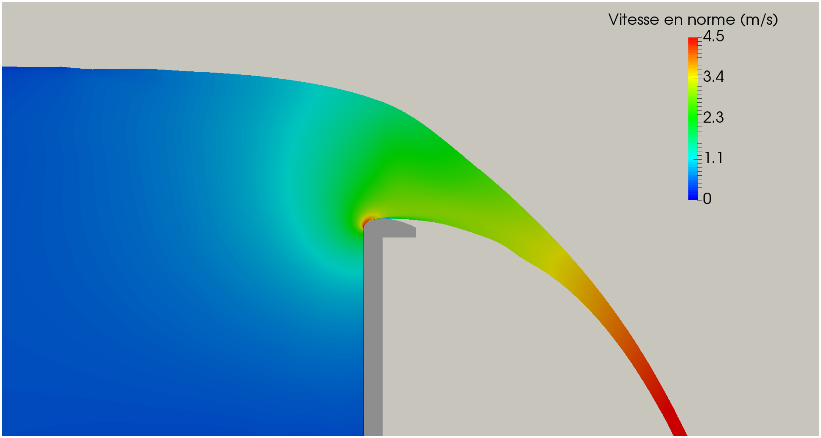


Figure: Slice view (middle plane along flow direction) - $H/H_D = 2.3$

Creager weir - 3D case

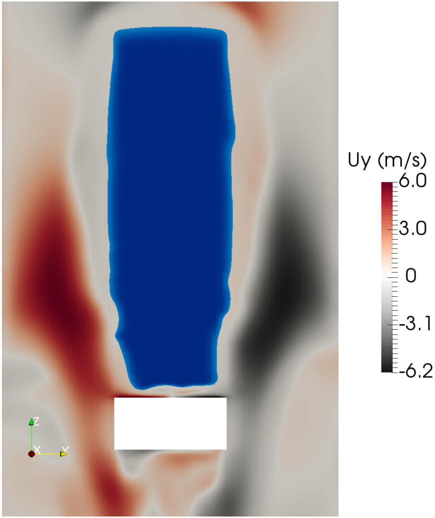


Figure: Slice with (y, z) plane at $x/H_D = 0.3$ - y velocity component - blue zone at the image center belongs to spilling flow - $H/H_D = 2.3$

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Piano Key (PK) Weir

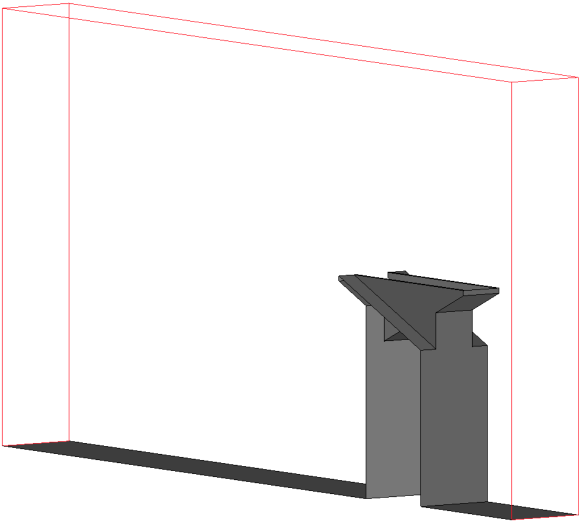


Figure: Geometry of a Piano Key Weir - height 4m

PK Weir

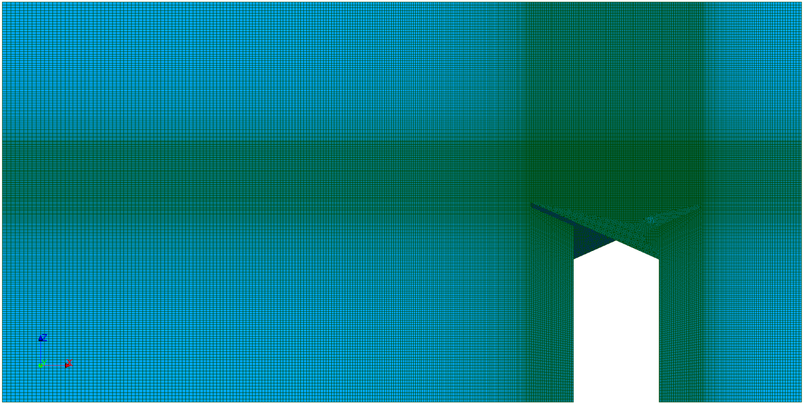
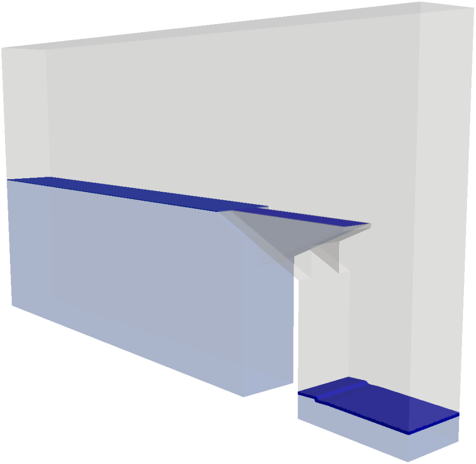


Figure: PK Weir - Mesh of the domain (min. cell size 5cm)

PK Weir

Simulation - Flow rate 15.6 m^2/s



PK Weir - comparison with height-flow rate law

Flow rate (m^2/s)	H_{exp} (m)	H_{simu} (m) $\pm 5cm$	Deviation %
3.5	0.5	0.55	10
8.2	1	1.2	20
15.6	2	2.3	15
22.4	3	3.3	10
25.5	3.5	3.9	11.4
35.5	5	5.1	2

Table: Comparison of the values of the height head H - experimental data : F. Lempérière

Conclusion

- Validation of the model for 2D overtopping flows.
- Low pressure area observed on Creager weirs and flow separation exhibited on 3D similations of Creager weir.
- Improvements needed on PK weir computations (robustness issues - ongoing work).

Perspectives

- Explore new designs of spilling weirs.
- Optimize existing shapes of weir $\rightarrow$ ongoing internship, workflow in Salome.
- For the VOF module, other topic foreseen: waves propagation and overflow.

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References

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