A two-dimensional code to simulate liquid film on the blades of steam turbines

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Code_Saturne user meeting
April 1, 2016

Outline

- Context and Objectives
- Liquid film model
- Simulations
- Conclusions and perspectives

Context and Objectives

<u>Wetness in steam turbine</u>

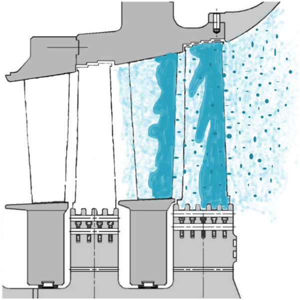
Nucleation

Growth

Deposition

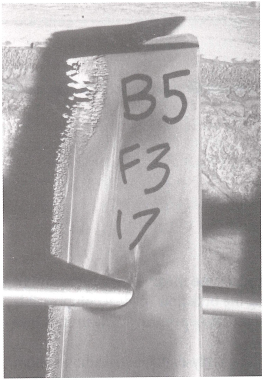
Liquid film

Atomization



Context and Objectives

Wetness induces **erosion** and **losses**

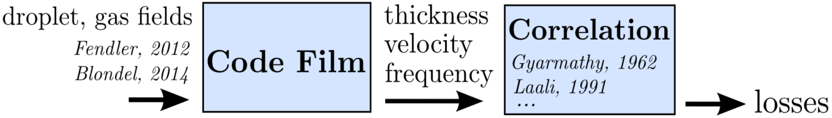


Hesketh, 2005, J. Mech. Eng. Sc.

Baumann's rule (1912): 1% humidity = 1% power loss

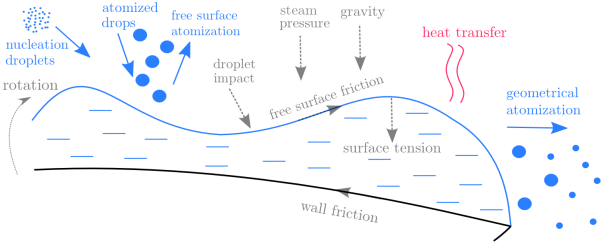
Context and **Objectives**

Describe the unsteady liquid film on stator and rotor blades with a model and numerical simulations



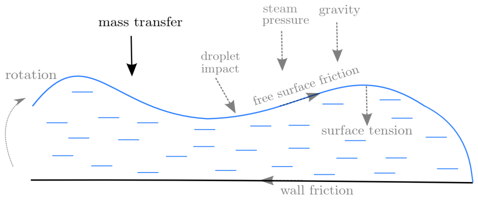
Liquid film model

Real phenomena



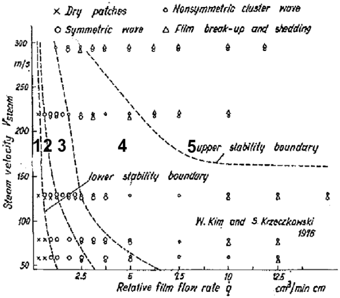
Model

$\nRightarrow$ curved surface



Liquid film model

⇒ Literature analysis: thin (10 to 100 μm), laminar, continuous



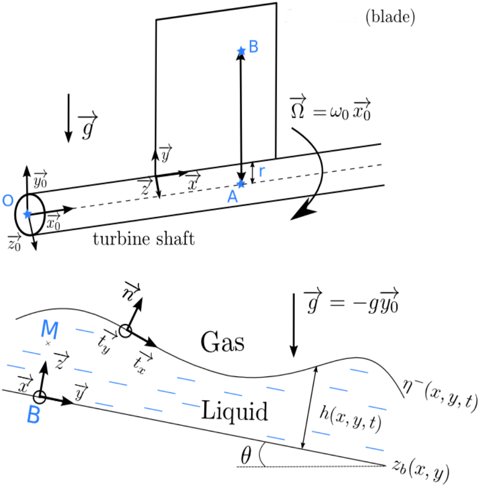
1. Rivulets and dry zones
2. Continuous film with 2d waves at the free surface
3. Transition
4. Continuous film with 3d waves at the free surface
5. Continuous film with 3d waves at the free surface and atomisation

Hammitt et al., 1981, Forschung im Ingenieurwesen

⇒ Modified Shallow-Water equations: integral formulation of simplified Navier-Stokes equations with specific physics

Liquid film model

Frames



Liquid film model

Simplified incompressible N-S equations at order $\mathcal{O}\left(\varepsilon^2\right)$ with $\varepsilon = h/l$.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} + \frac{\partial uw}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{1}{\rho}\frac{\partial \tau_{xz-r}}{\partial z}$$

$$\frac{\partial v}{\partial t} + \frac{\partial uv}{\partial x} + \frac{\partial v^2}{\partial y} + \frac{\partial vw}{\partial z} = g_y - \frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{1}{\rho}\frac{\partial \tau_{yz-r}}{\partial z} + \omega_0^2(r+y) + 2\omega_0 w$$

$$\frac{\partial p}{\partial z} = \rho g_z - 2\rho\omega_0 v + \rho\omega_0^2 z$$

with $\tau_{xz-r} = \mu\frac{\partial u}{\partial z}$ and $\tau_{yz-r} = \mu\frac{\partial v}{\partial z}$

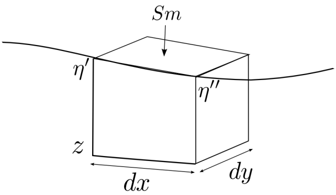
Liquid film model

Boundary conditions

- At the wall:

$u_{|_{z=z_b}} = 0;\ v_{|_{z=z_b}} = 0;\ w_{|_{z=z_b}} = 0$

- At the free surface:



$$\text{mass balance} \Rightarrow \frac{dm}{dt} = \rho S_m dxdy$$

$$\Rightarrow \frac{d(\eta - z)}{dt} = S_m$$

$$\Rightarrow w_{|_{z=\eta}} = \partial_t \eta + u_{|_{z=\eta}} \partial_x \eta + v_{|_{z=\eta}} \partial_y \eta - S_m$$

Liquid film model

Exact form of the integration over the thickness of the film

$$\frac{\partial h}{\partial t} + \frac{\partial h\overline{u}}{\partial x} + \frac{\partial h\overline{v}}{\partial y} = S_m$$

$$\frac{\partial h\overline{u}}{\partial t} + \frac{\partial}{\partial x}\left(\int_{z_b}^{\eta^-} u^2 dz + \frac{gcos(\theta)h^2}{2}\right) + \frac{\partial}{\partial y}\int_{z_b}^{\eta^-} uvdz + 2\omega_0 \int_{z_b}^{\eta^-} \frac{\partial \int_z^{\eta^-} vdz}{\partial x} dz - \frac{\omega_0^2}{2} h \frac{\partial \eta^2}{\partial x} =$$
$$-\frac{h}{\rho}\frac{\partial \mathscr{P}_{|_{z=\eta^-}}^{liquid}}{\partial x} - hgcos(\theta)\frac{\partial z_b}{\partial x} + \frac{1}{\rho}\left(\tau_{xz-r}|_{\eta^-} - \tau_{xz-r}|_{z_b}\right) + u_{|_{\eta^-}} S_m$$

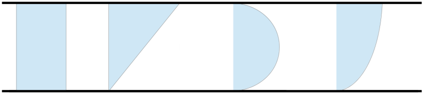
$$\frac{\partial h\overline{v}}{\partial t} + \frac{\partial}{\partial x}\int_{z_b}^{\eta^-} uvdz + \frac{\partial}{\partial y}\left(\int_{z_b}^{\eta^-} v^2 dz + \frac{gcos(\theta)h^2}{2}\right) + 2\omega_0 \int_{z_b}^{\eta^-} \frac{\partial \int_z^{\eta^-} vdz}{\partial y} dz - \frac{\omega_0^2}{2} h \frac{\partial \eta^2}{\partial y} =$$
$$g_y h - \frac{h}{\rho}\frac{\partial \mathscr{P}_{|_{z=\eta^-}}^{liquid}}{\partial y} - hgcos(\theta)\frac{\partial z_b}{\partial y} + \frac{1}{\rho}\left(\tau_{yz-r}|_{\eta^-} - \tau_{yz-r}|_{z_b}\right) + 2\omega_0 h\overline{w} + h\omega_0^2(r+y) + v_{|_{\eta^-}} S_m$$

$\Rightarrow$ 8 required closure laws (colored terms)

Liquid film model

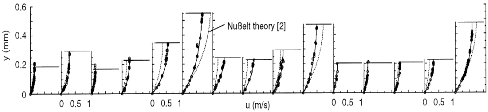
Closure laws : $\int_{z_b}^{\eta^-} u^2 dz, \int_{z_b}^{\eta^-} uv dz, \int_{z_b}^{\eta^-} v^2 dz$

$$\int_{z_b}^{\eta^-} u^2 dz = \Gamma h \overline{u}^2 \text{ with } \overline{u} = \frac{1}{h}\int_{z_b}^{\eta^-} u dz$$



Legitimacy for parabolic velocity profile:

- Theory: Poiseuille flow
- Experiments: Falling films [1,2,3]



Adomeit and Reiz, 2000, IJMF

For the model, $\Gamma = 1$ or $\Gamma = 6/5$

[1] Bertshy et al., 1983, *JFM*

[2] Alekseenko, 1985, *AIChE Journal*

[3] Adomeit, 2000, *IJMF*

Liquid film model

Closure laws : $u_{|_{\eta^-}}$, $v_{|_{\eta^-}}$

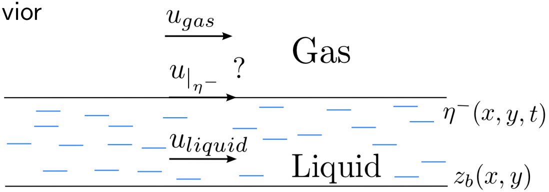
$\Rightarrow$ Entropy equation for a global system composed with liquid and vapor

Stating that:

- the velocity at the free surface depends on both phases
- the entropy of the system must increase in time
- the transfer of mass between the two phases is only due to thermodynamic phenomena
- the velocity of the liquid is represented by its mean velocity

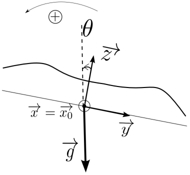
the only entropy-consistent velocity is: $u_{|_{\eta^-}} = \frac{\overline{u} + u_{vapor}}{2}$

$\Rightarrow$ Physical behavior



Liquid film model

General model



$$\frac{\partial h}{\partial t} + \frac{\partial h\overline{u}}{\partial x} + \frac{\partial h\overline{v}}{\partial y} = S_m$$

$$\frac{\partial h\overline{u}}{\partial t} + \frac{\partial\left(\Gamma h\overline{u}^2 + \frac{gcos(\theta)h^2}{2}\right)}{\partial x} + \frac{\partial \Gamma h\overline{u}\ \overline{v}}{\partial y} + 2\omega_0\left(\overline{v}h\frac{\partial \eta}{\partial x} + \frac{h^2}{2}\frac{\partial \overline{v}}{\partial x}\right) - \frac{\omega_0^2}{2}h\frac{\partial \eta^2}{\partial x} =$$

$$-\frac{h}{\rho}\frac{\partial \mathscr{P}_{|_{z=\eta-}}^{film}}{\partial x} - hgcos(\theta)\frac{\partial z_b}{\partial x} + \frac{1}{\rho}\left(\tau_{xz-r}{}_{|_\eta} - \tau_{xz-r}{}_{|_{z_b}}\right) + \left(\frac{\overline{u} + u_{gas}}{2}\right)S_m$$

$$\frac{\partial h\overline{v}}{\partial t} + \frac{\partial \Gamma h\overline{u}\ \overline{v}}{\partial x} + \frac{\partial\left(\Gamma h\overline{v}^2 + \frac{gcos(\theta)h^2}{2}\right)}{\partial y} + 2\omega_0\left(\overline{v}h\frac{\partial \eta}{\partial y} + \frac{h^2}{2}\frac{\partial \overline{v}}{\partial y}\right) - \frac{\omega_0^2}{2}h\frac{\partial \eta^2}{\partial y} + 2\omega_0\frac{h^2}{2}\left(\frac{\partial \overline{u}}{\partial x} + \frac{\partial \overline{v}}{\partial y}\right) = gsin\theta h -$$

$$\frac{h}{\rho}\frac{\partial \mathscr{P}_{|_{z=\eta-}}^{film}}{\partial y} - hgcos(\theta)\frac{\partial z_b}{\partial y} + \frac{1}{\rho}\left(\tau_{yz-r}{}_{|_\eta} - \tau_{yz-r}{}_{|_{z_b}}\right) + h\omega_0^2(r+y) + 2\omega_0 h\left(\overline{u}\frac{\partial z_b}{\partial x} + \overline{v}\frac{\partial z_b}{\partial y}\right) + \left(\frac{\overline{v} + v_{gas}}{2}\right)S_m$$

	Stator model	**Rotor model with $\Gamma = 1$**
Hyperbolicity	$cos\theta > 0$	$gcos\theta + 2\omega_0\overline{v} - \omega_0^2 z_b > 0$
Entropy-entropy flux couple	$\Gamma = 1$	yes
Convex entropy	$cos\theta > 0$	$gcos\theta - \omega_0^2\eta > 0$
Galilean invariance	$\Gamma = 1$	yes
Rotational invariance	yes	no (Coriolis)

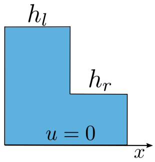
Simulations - verification

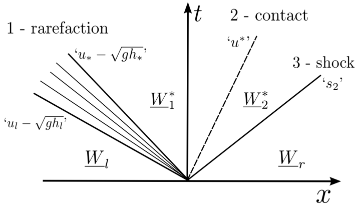
Dam break (particular Riemann problem)

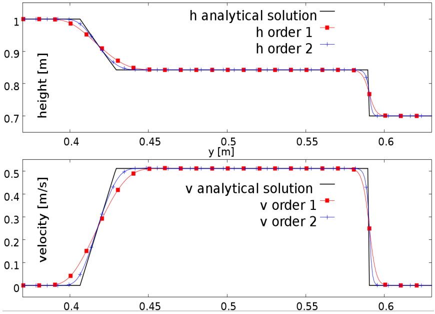
$$(h)_{,t} + (h\overline{u})_{,x} = 0$$

$$(h\overline{u})_{,t} + \left(h\overline{u}^2 + \frac{g\cos(\theta)h^2}{2}\right)_{,x} = 0$$

$$(h\overline{v})_{,t} + (h\overline{u}\ \overline{v})_{,x} = 0$$







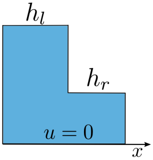
Simulations - verification

Dam break with $\Gamma = 6/5$

$$(h)_{,t} + (h\overline{u})_{,x} = 0$$

$$(h\overline{u})_{,t} + \left(\Gamma h\overline{u}^2 + \frac{g\cos(\theta)h^2}{2}\right)_{,x} = 0$$

$$(h\overline{v})_{,t} + (\Gamma h\overline{u}\ \overline{v})_{,x} = 0$$



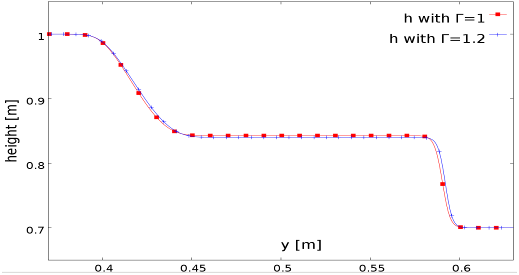
$$\lambda_1 = \Gamma U_n - c\sqrt{1 + \Gamma(\Gamma - 1)M^2}$$

$$\lambda_2 = \Gamma U_n$$

$$\lambda_3 = \Gamma U_n + c\sqrt{1 + \Gamma(\Gamma - 1)M^2}$$

with
$U_n = \overrightarrow{n_x}\overline{u} + \overrightarrow{n_y}\overline{v}$
$M = \dfrac{U_n}{\sqrt{g\cos(\theta)}}$

$\Rightarrow$ No Riemann invariant found, so no analytical solution found.

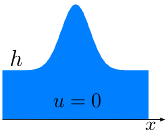


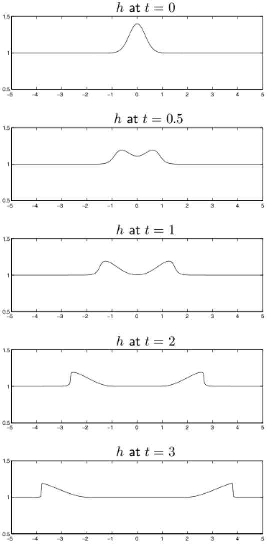
Simulations - verification

Hump (particular Riemann problem)

$$(h)_{,t} + (h\bar{u})_{,x} = 0$$

$$(h\bar{u})_{,t} + \left(h\bar{u}^2 + \frac{g cos(\theta) h^2}{2}\right)_{,x} = 0$$





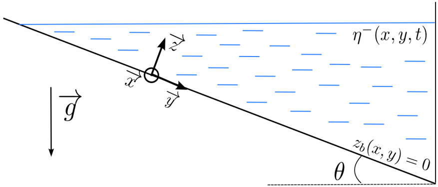
Leveque, 2004

Simulations - verification

Inclined lake at rest

$$(h)_{,t} + (h\overline{v})_{,y} = 0$$

$$(h\overline{v})_{,t} + \left(h\overline{v}^2 + \frac{gcos(\theta)h^2}{2}\right)_{,y} = ghsin(\theta)$$



The inclined lake at rest is an analytical solution of the model ($\overline{v} = 0$ and $h = tan\theta + h_0$). The code verify this solution as the initial condition do not evolve.

Simulations - validation

Falling liquid film on an inclined plate

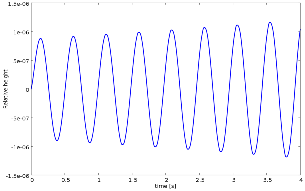
- A small perturbation is amplified (unstable) if $Re_{film} > Re_c$.
  Theory : Linear stability analysis of N-S equations
  Experiment : Liu and Gollub, 1994, *PoF*
  $\Rightarrow \quad Re_c = \frac{5}{6} cot(\theta)$

- Model $(h)_{,t} + (h\overline{v})_{,y} = 0$

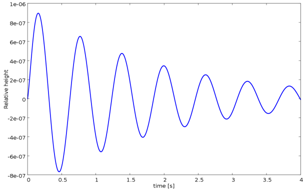
$$(h\overline{v})_{,t} + \left(h\overline{v}^2 + \frac{gcos(\theta)h^2}{2}\right)_{,y} = ghsin(\theta) - \frac{3\nu\overline{v}}{h}$$

- Relative height at the center versus time for $\theta = 6.4 \Rightarrow Re_c = 7.4$

$Re_{film} = 7.9$
($> Re_c$)



$Re_{film} = 7.0$
($< Re_c$)



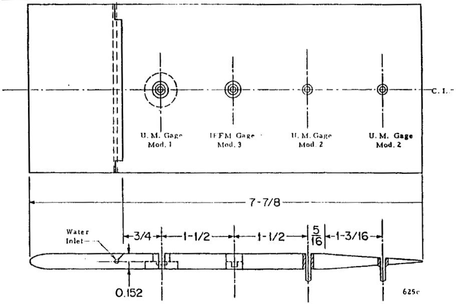
Cells number/Re	7.7	7.8	7.9	8.0	8.1	8.2	8.3
100	S	S	S	S	S	S	U
1000	S	S	U	U	U	U	U
10000	S	U	U	U	U	U	U
100000	S	U	U	U	U	U	U

S for stable
U for unstable

Simulations - validation

Highly sheared film on a plate under steam turbine conditions

- Experiment [4]: 0.2 bar, 52.2 °C, steam velocity 60-390 m/s, film flow rate $5.10^{-7} m^3/s$
  $\Rightarrow$ Measured film height from 50 to 200 μm



- Model

$$(h)_{,t} + (h\overline{v})_{,y} = S_m$$

$$(h\overline{v})_{,t} + \left(h\overline{v}^2 + \frac{g cos(\theta) h^2}{2}\right)_{,y} = g sin(\theta) h + \frac{1}{\rho}\left(\tau_{yz-r}\big|_{\eta} - \tau_{yz-r}\big|_{z_b}\right)$$

[4] Hammitt et al., 1975, *Report UMICH and 1981, Forschung im Ingenieurwesen*

Simulations — Highly sheared film on a plate under steam turbine conditions

with wall shear

(a) - $\frac{3\mu}{h}\overline{v} - \frac{\tau_{yz-r}|_{\eta}}{2}$

(b) - $\frac{c_f \rho |\overline{v}| \overline{v}}{2}$ with $c_f = \frac{16}{Re}$

(c) - $\frac{c_f \rho |\overline{v}| \overline{v}}{2}$ with $c_f = \frac{24}{Re}$

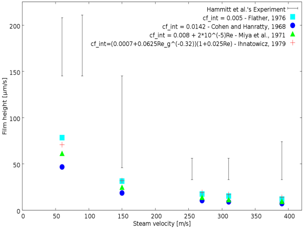
with free surface shear $\frac{c_{f\_int} \rho |u_g - \overline{u}| (u_g - \overline{u})}{2}$

(1) - $c_{f\_int} = 0.005$

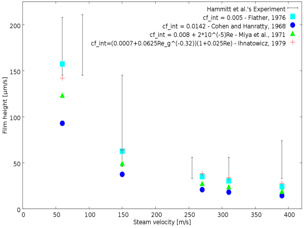
(2) - $c_{f\_int} = 0.0142$

(3) - $c_{f\_int} = 0.008 + 2 \times 10^{-5} Re$

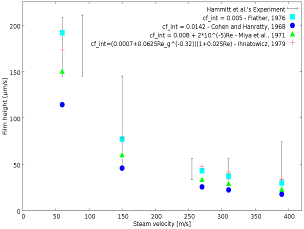
(4) - $c_{f\_int} = \left(0.0007 + 0.0625 Re_g^{-0.32}\right)(1 + 0.025 Re)$



Wall shear (a)



Wall shear (b)



Wall shear (c)

Simulations

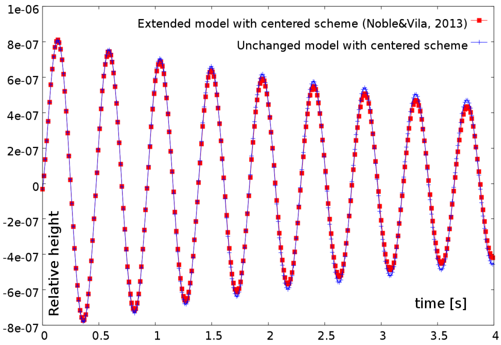
Implementation of the surface tension ($\frac{\sigma}{\rho} h \partial_{yyy} h$)

Option 1:

- unchanged model
- centered scheme

Option 2 (Noble & Vila, 2013):

- extended model (+ 1 equation which reduced the order of the system)
- centered scheme



Model (option 1):

$$(h)_{,t} + (h\bar{v})_{,y} = 0$$

$$(h\bar{v})_{,t} + \left(h\bar{v}^2 + \frac{g cos(\theta) h^2}{2}\right)_{,y} = g h sin(\theta) - \frac{3\nu\bar{v}}{h} + \frac{\sigma}{\rho} h \partial_{yyy} h$$

Falling film: linear stability analysis
$Re_{film} = 8.26$, $\theta = 6.4^\circ$ and 100 nodes

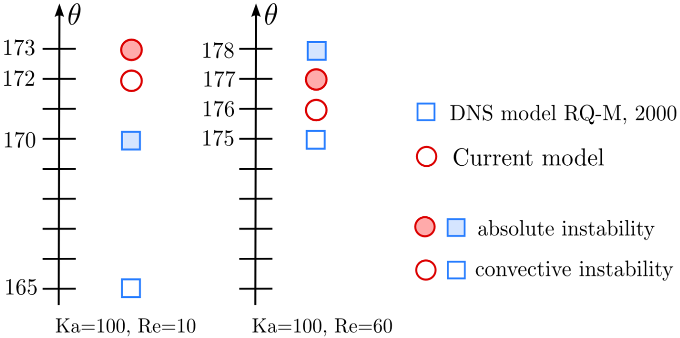
Simulations - validations

Inverted gravity

A liquid film flowing down the bottom side of an inclined plate is always unstable. This instability can be absolute or convective depending on the Kapitza number (surface tension/inertia), the Reynolds number (viscosity/inertia) and the angle θ. *Kofman, PoF, 2016*

$$(h)_{,t} + (h\overline{v})_{,y} = 0$$

$$(h\overline{v})_{,t} + \left(h\overline{v}^2 + \frac{g cos(\theta) h^2}{2}\right)_{,y} = g h sin(\theta) - \frac{3\nu\overline{v}}{h} + \frac{\sigma}{\rho} h \partial_{yyy} h$$



Conclusions and perspectives

- A model (stator and rotor) for liquid film in steam turbines has been proposed and its properties have been examined.

- The entire stator model has been implemented (convection, mass transfer, shear at the wall and at the free surface, surface tension, gas pressure, droplet impact momentum)

- The stator model has been verified with two Riemann problems, the inclined lake at rest and validated with the falling film, the highly sheared film and the inverted gravity experiments.

- Implementation of rotational effect.

- Chaining method (Fendler and Blondel's results)

Thank you for your attention.