Implementation of an aeroacoustic module based on Ffowcs Williams & Hawkings acoustic analogy in *Code_Saturne*

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Introduction

Noise prediction is an important part of many engineering applications ranging from aerospace (particularly for rotor noise generated by helicopters and turboprops) to combustion, to car manufacture industry, to noise generated by electrical appliances. Noise can be directly obtained from the compressible Navier-Stokes equations through Direct Numerical Simulation (DNS). However, DNS is a very computationally demanding approach to fluid simulations where all length scales of the turbulent spectrum have to be resolved. For this reason DNS is still limited to moderate Reynolds numbers and relatively simple geometries. Noise predictions further increase the computational requirements since noise levels might be required very far away from the noise source locations. As a consequence, very large computational domains, which might not be necessary from a pure hydrodynamic point of view, are mandatory.

A very well established solution to reduce computational demand is to separate the computation of the noise sources, which can be performed using several CFD methodologies, from the propagation of the sound itself and assume there is no feedback in the noise generation. This assumption is well verified in the case of subsonic flows (i.e. Mach number $M < 1$) and when the noise sources are compact (i.e. confined to a limited region) with the sound propagating into a fluid at rest. This procedure for modelling noise is referred to as hybrid computational aeroacoustic (CAA) and the theory is known as the aeroacoustic analogy.)

FW-H Formulation

The FW-H equation is the most general form of the Lighthill's acoustic analogy (Lighthill, 1952) and can be obtained as an exact rearrangement of the continuity and Navier-Stokes equations. The method is based on the definition of an arbitrary control surface $f = 0$ and the fluid enclosed (i.e. $f < 0$) can be represented with fluid at rest plus a distribution of sources at $f = 0$ (see Fig. 1). The differential form of the FW-H equation takes the form of an inhomogeneous wave equation and reads:

$$\frac{1}{c^2}\frac{\partial^2 p'}{\partial t} - \nabla^2 p' = \Box^2 p' = \frac{\partial^2 T_{ij} H(f)}{\partial x_i \partial x_j} - \frac{\partial L_i \delta(f)}{\partial x_i} + \frac{\partial \rho_0 U_i \hat{n}_i \delta(f)}{\partial t}$$

with

$$H(f) = \begin{cases} 1 & f > 0 \\ 0 & f \leq 0 \end{cases} \qquad T_{ij} = \rho u_i u_j - \tau_{ij} + \left(p' - c^2 \rho'\right)\delta_{ij}$$

$$L_i = P_{ij}\hat{n}_j + \rho u_i \left(u_n - v_n\right)$$

$$\delta(f) = \begin{cases} 1 & f = 0 \\ 0 & f \neq 0 \end{cases} \qquad U_i = \left(1 - \frac{\rho}{\rho_o}\right) v_i + \frac{\rho u_i}{\rho_0}$$

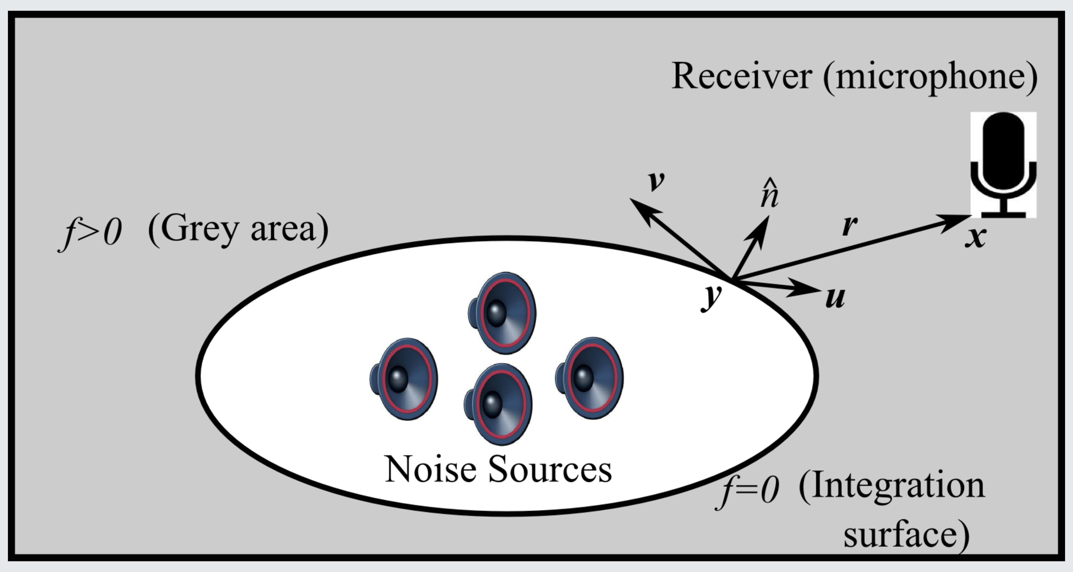

Figure 1: Schematic of the FW-H analogy

An integral formulation of the solution has been proposed by Farassat and Succi (1982), where the acoustic pressure fluctuations p' can be expressed as the summation of the three different contributions:

$$p' = p'_L + p'_T + p'_Q$$

with p'_L being the loading noise, p'_T the thickness noise and p'_Q the quadrupole noise. The analytical definition of the different terms reads:

$$4\pi p'_T = \int_{f=0} \left[\frac{\rho_0\left(\dot{U}_n + U_{\dot{n}}\right)}{r\left(1 - M_r\right)^2}\right]_{ret} dS + \int_{f=0} \left[\frac{\rho_0 U_n \left(r\dot{M}_r + c\left(M_r - M^2\right)\right)}{r^2\left(1 - M_r\right)^3}\right]_{ret} dS$$

$$4\pi p'_L = \frac{1}{c}\int_{f=0} \left[\frac{\dot{L}_r}{r\left(1 - M_r\right)^2}\right]_{ret} dS + \int_{f=0} \left[\frac{L_r - L_m}{r^2\left(1 - M_r\right)^2}\right]_{ret} dS + \frac{1}{c}\int_{f=0} \left[\frac{L_r\left(r\dot{M}_r + c\left(M_r - M^2\right)\right)}{r^2\left(1 - M_r\right)^3}\right]_{ret} dS$$

$$4\pi p'_Q = \int_{f>0} \left[\frac{K_1}{c^2 r} + \frac{K_2}{c r^2} + \frac{K_3}{r^3}\right]_{ret} dV$$

The $\dot{}$ stands for a time derivative (i.e. $\dot{L} = \partial L/\partial t$) and the subscript r indicates the projection of a vector in the $\hat{r}_i$ direction (i.e $L_r = L_i \cdot \hat{r}_i$).

Advance time formulation

The retarded time formulation presented above implies that the formulation is expressed in terms of reception time at the receiver. This implies that for a given receiver time t the disturbances are emitted at different times t_{ret} depending on the relative position between the receiver and the source.

$$t_{ret} = t - \frac{\|\mathbf{x}(t) - \mathbf{y}(t_{ret})\|}{c} = \frac{r(t_{ret})}{c}$$

This approach has the disadvantage that for a given time t the aeroacoustic simulation has to access to different retarded times and corresponding to several instances of the CFD simulations, that have to be keep in memory or on disk.

A solution to the problem is to adopt an advance in time formulation as proposed by Casalino (2002). In this case the computation time for the acoustic is the emission time and the time at which the disturbance is reaching the receiver is computed using:

$$t_{adv} = t + \mathcal{T} = t + \frac{r(t)}{c}$$

With this formulation only one instances of the CFD is necessary and the memory requirement is transferred to the receiver. This generally is not an issue since the number of receivers is generally much smaller that the size of the CFD mesh.

Scatter of a plane wave by a rigid cylinder

A first analytical test case to validate the method is the scatter of a plane wave impinging on a rigid cylinder. The resulting pressure is function of the time, the distance from the centre of the cylinder and the angle mesured from the direction of the incoming wave and reads:

$$p'(r, t, \theta) \simeq -P\sqrt{\frac{D}{\pi r}}\psi_s(\theta)\exp\left(ik(r - ct)\right)$$

with

$$\psi_s(\theta) = \sqrt{\frac{2}{kD}}\sum_{m=0}^{\infty}\varepsilon_m \sin(\gamma_m)\exp(-i\gamma_m)\cos(m\theta)$$

The control surface is positioned on the surface of the rigid cylinder for which an analytical definition of the pressure is also available (Morse and Ingard, 1968).

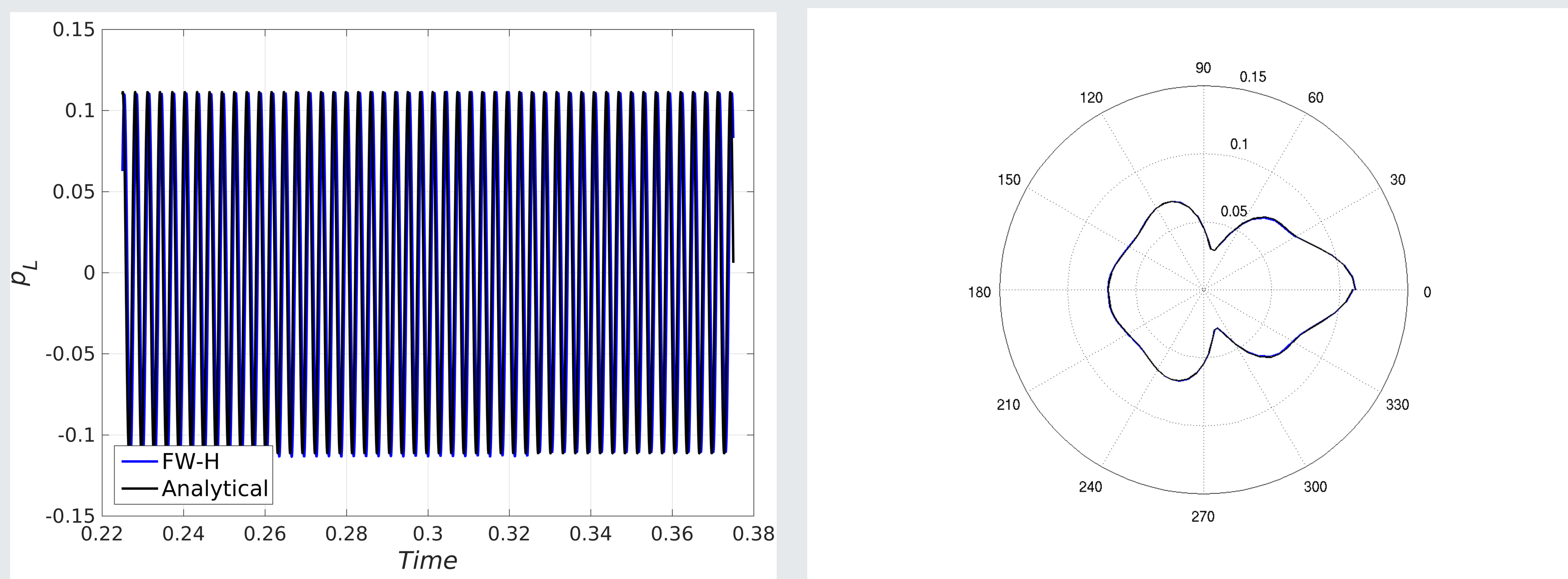

Figure 2: Comparison of the part of p' at $\theta = 0$ (left). Directivity pattern (right)

Fig. 2 on the left shows the comparison between the analytical and numerical solution at the receiver located at angular position $\theta = 0$, whereas on the right it is reported the directivity pattern around the cylinder. The comparison is very good for all angular locations, and only a small phase lag is noticeable between the two solutions.

Conclusion and future work

An aeroacoustic module is under development in *Code_Saturne* and preliminary results seems encouraging. Additional test cases based on flow around a cylinder using the compressible module are on-going.

To improve the memory requirement and management of the acoustic module a solution using code coupling is also under investigation.

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Acknowledgements

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