Evaluation of water wave modeling with an arbitrary Lagrangian-Eulerian approach

Jeffrey C. Harris

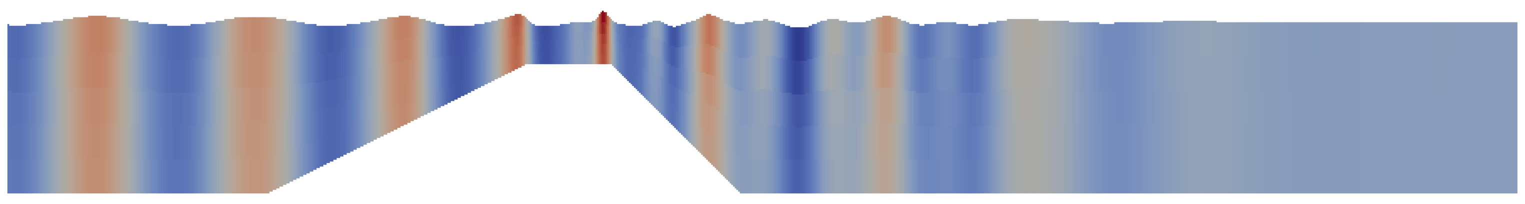
Saint-Venant Hydraulics Laboratory, Université Paris-Est (EDF R&D, Cerema, ENPC)

jeffrey.harris@enpc.fr

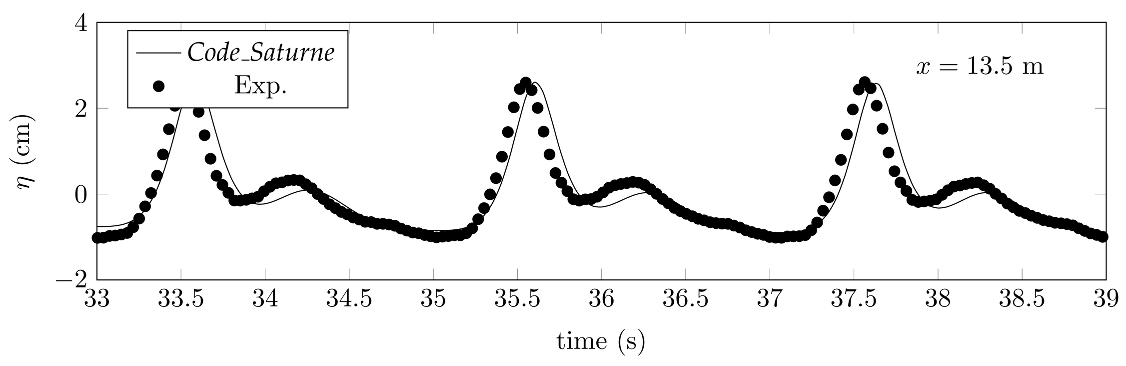
Introduction

For reasons of efficiency and accuracy, water wave propagation is often modeled with potential (inviscid) models rather than Navier-Stokes solvers, but for some wave-induced flows, such as wave-structure interaction, viscous effects are important. General purpose models can have limitations when applied to such free-surface problems when dealing with large amplitude waves, run-up, or propagation over long distances. Here we test the capabilities of *Code_Saturne* for modeling water wave propagation and wave-induced flows with the ALE (Arbitrary Lagrangian-Eulerian) module, following earlier work of Teles et al. (2013).

Wave propagation over a submerged bar



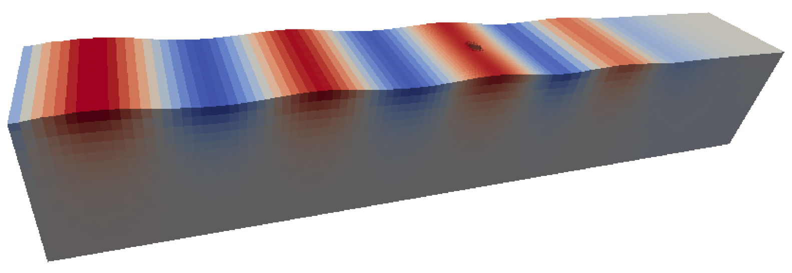
A common test of non-hydrostatic wave models is propagation over a submerged bar, which is beyond the range of a shallow-water assumption, or even some Boussinesq models. We are able to match the experimental results of Beji and Battjes (1993), comparing against time-series measured at various wave gauges:



In this test case, 38522 cells are used to discretize the domain. On the left side, the velocity field is imposed from Airy wave theory, plus a Stokes drift term, as in Ma et al. (2012), to ensure mass conservation. Waves are absorbed on the right side through a surface pressure proportional to the vertical velocity at the surface, as in Grilli and Horrillo (1997).

Vertical cylinder in regular waves

As a more complex test, we can consider the interaction of regular Stokes waves with a surface-piercing structure such as a monopile. Comparing with the experiments of Huseby and Grue (2000) for certain conditions ($kr = 0.245$ and $kA = 0.1$), we can consider various harmonics of the wave forces on a vertical cylinder:



	Code_Saturne	Exp.
$\lvert F_1\rvert/(\rho g A r^2)$	6.44	6.42
$\lvert F_2\rvert/(\rho g A^2 r)$	0.41	0.41
$\lvert F_3\rvert/(\rho g A^3)$	0.50	0.28

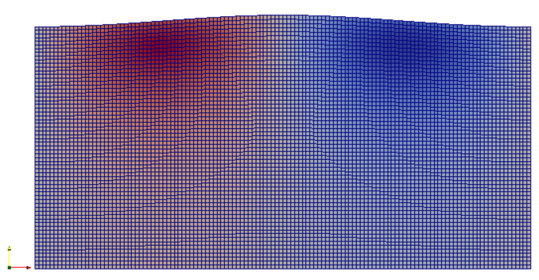
The present results for the higher-order terms are noisy, so the harmonic analysis is computed over five periods of oscillation, after ten periods of model spin-up. Extensions of this setup to gravity-based foundations or other practical applications with irregular waves is straightforward. Tests with moving surface-piercing objects, however, can lead to a breakdown of the existing ALE module. Investigations into grid convergence and larger incident waves are ongoing.

Acknowledgements

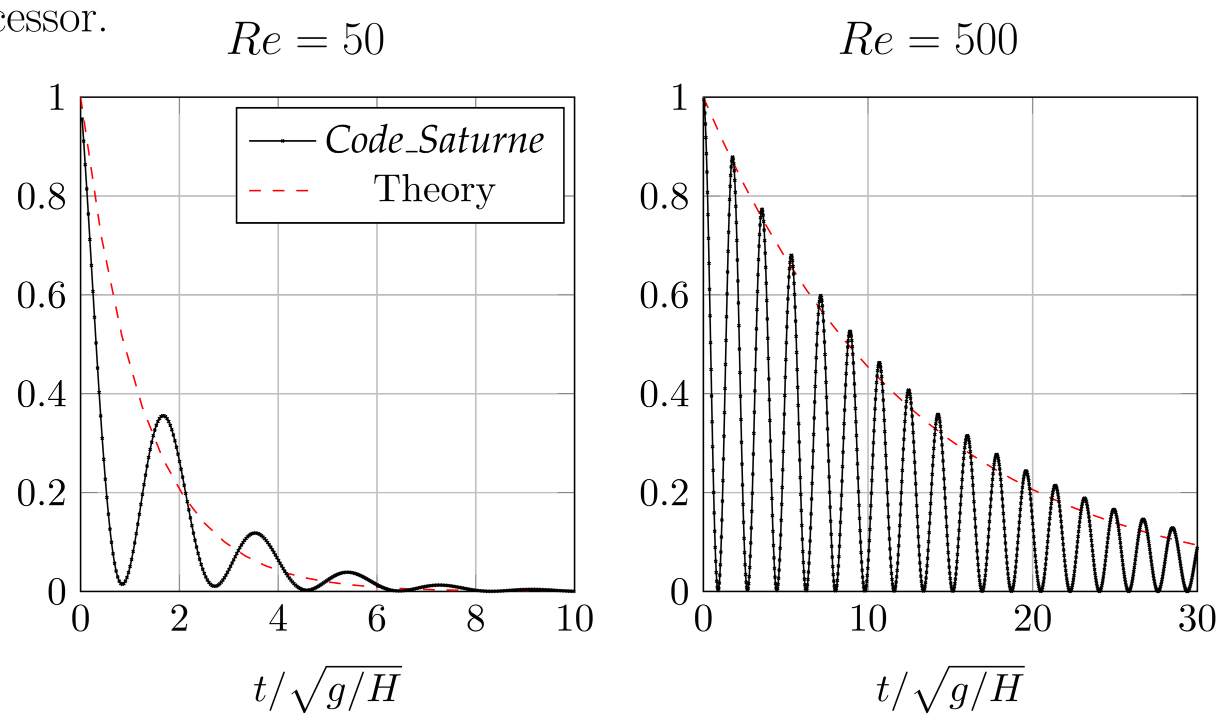
The author would like to thank Sébastien Boyaval, Martin Ferrand, and Michel Benoit for helpful discussions.

Viscous decay of standing waves

To validate the viscous component of the wave modeling, like Antuono and Colagrossi (2013), we can consider the case of a small amplitude periodic standing wave. We impose on an undisturbed domain the velocity field of an Airy wave solution, and examine the evolution of the kinetic energy compared to the expected exponential decay. These results are with a two-dimensional grid, 128×64, taking a few minutes on a single processor.



$$\mathcal{E}_K = \int \frac{\|\mathbf{u}\|^2}{2} dV$$

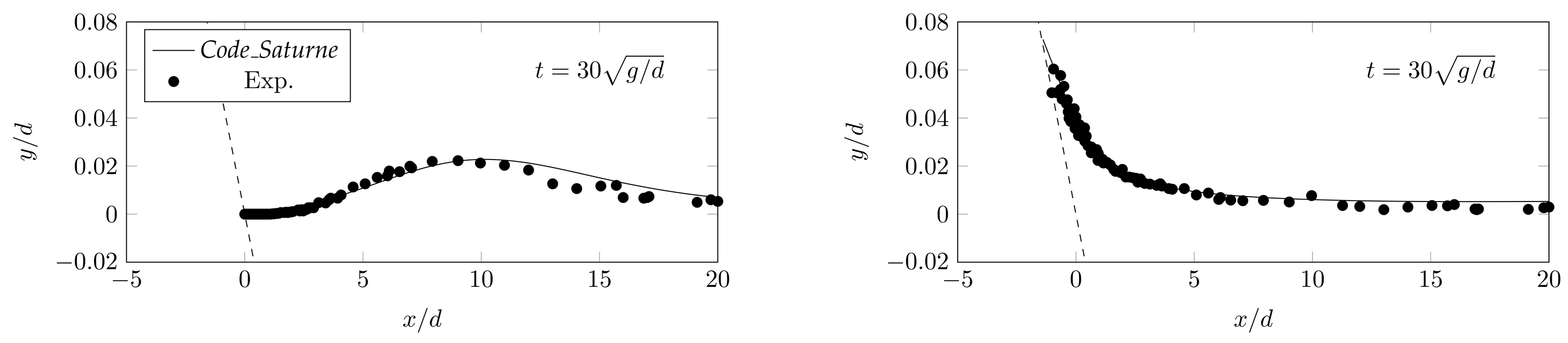


Varying the Reynolds number for various periodic wave propagation tests in deep water, we roughly match theory. Additional tests have shown the differences in viscous decay in intermediate or shallow water, the effect of slip or no-slip boundary conditions, and the decay of enstrophy over time.

This is also the subject of upcoming work with Sébastien Boyaval for convergence study and comparison with standard Taylor-Hood finite element discretizations.

Non-breaking solitary wave run-up

Wave run-up can be challenging for some models, but on a simple beach with a slope of $\cot\beta = 19.85$, and a solitary wave amplitude $H/d = 0.0185$, wave profiles and run-up match the experiments of Synolakis (1987) for non-breaking solitary waves:



Summary

From numerous benchmarks, we see that *Code_Saturne* can be applied to standard coastal wave modeling problems, capture non-hydrostatic effects, and be used for complex geometries. Due to limitations of the present ALE formulation, the modeling of floating surface-piercing bodies or wave breaking will require further investigation.

References

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